CSC236 tutorial exercises, Week #9
best before Thursday evening

These exercises are intended to give you some practice applying the Master Theorem[1] to algorithm design.

1. Consider the following sketch of a divide-and-conquer algorithm $r(s)$ for reversing a string:

   (a) s is a string.

   (b) If $\text{len}(s) < 2$, return s

   (c) Else, partition s into three roughly equal parts: prefix s_1, suffix s_3, and mid-section s_2, and return $r(s_3) + r(s_2) + r(s_1)$.

   (d) You may assume that the time complexity of string concatenation of $s_3 + s_2 + s_1$ is proportional to $\text{len}(s_3) + \text{len}(s_2) + \text{len}(s_1)$

   Use the Master Theorem to find the asymptotic time complexity of function r in terms of $\text{len}(s)$. Be sure to show all the components of your analysis, including the values of a, b, and d. How does this compare to the complexity of simply copying the string elements in reverse order, using a loop?

2. Describe a ternary version of MergeSort where the list segment to be sorted is divided into three (roughly) equal sub-lists, rather than two. Use the Master Theorem to find the asymptotic time complexity of your ternary MergeSort in terms of the length of the list segment being sorted, and compare/contrast it with the version we analyzed in class. Be sure to show all the components of your analysis, including the values of a, b, and d.

3. Consider the following sketch of bisection algorithm $bis(f, a, b, \gamma, \delta)$ to approximate a root of a function:

   (a) $f : \mathbb{R} \mapsto \mathbb{R}$ is a function, $a, b \in \mathbb{R}$ with $f(b) \times f(a) \leq 0$, $\gamma, \delta \in \mathbb{R}^+$

   (b) If $|b - a| < \gamma$ return $(a + b)/2$.

   (c) If $|f(a)| < \delta$ return a.

   (d) If $|f(b)| < \delta$ return b.

   (e) If $f(a) \times f([a + b]/2) \leq 0$ return $bis(f, a, (a + b)/2, \gamma, \delta)$.

   (f) Otherwise return $bis(f, (a + b)/2, b, \gamma, \delta)$

   Use the Master Theorem to find the asymptotic time complexity of function bis in terms of $|b - a|/\gamma$. Be sure to show all the components of your analysis.

[1] Very abbreviated version on next page...

$$T(n) = \begin{cases} k & \text{if } n \leq b \\ a_1 T(\lceil n/b \rceil) + a_2 T(\lfloor n/b \rfloor) + f(n) & \text{if } n > b \end{cases}$$

$$T(n) \in \begin{cases} \theta(n^d) & \text{if } a < b^d \\ \theta(n^d \log_b n) & \text{if } a = b^d \\ \theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$