CSC236 tutorial exercises, Week #4
best before Thursday evening

These exercises are intended to give you practice with structural and induction and well-ordering.

1. Use the Principle of Well Ordering and contradiction to prove that there are no positive integers x, y, z such $x^3 + 3y^3 = 9z^3$.

2. Define the set of expressions $\mathcal{E}$ as the smallest set such that:

   (a) $x, y, z \in \mathcal{E}$.

   (b) If $e_1, e_2 \in \mathcal{E}$, then so are $(e_1 + e_2)$ and $(e_1 \times e_2)$.

   Define $s_1(e)$: Number of symbols from $\{(,), +, \times\}$ in e, counting duplicates.

   Define $s_2(e)$: Number of symbols from $\{x, y, z\}$ in e, counting duplicates.

   Use structural induction to prove that for all $e \in \mathcal{E}$, $s_1(e) = 3(s_2(e) - 1)$.

3. Define the set of non-empty full binary trees, $\mathcal{T}$, as the smallest set such that:

   (a) Any single node is an element of $\mathcal{T}$.

   (b) If $t_1, t_2 \in \mathcal{T}$, n is a node that belongs to neither t_1 nor t_2, and t_1, t_2 have no nodes in common, then n together with edges to t_1 and t_2 is also an element of $\mathcal{T}$.

   Use structural induction to prove that any non-empty full binary tree has exactly one more leaf than internal nodes.