CSC236 fall 2018

complete induction

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use Introduction to the Theory of Computation, Section 1.3

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Outline

Principle of complete induction

Examples of complete induction

Complete Induction

another flavour needed

Every natural number greater than 1 has a prime factorization

Try some examples

How does the factorization of 8 help with the factorization of 9?

notational convenience...

I will use (though you don't have to) the following:

$$\bigwedge_{k=0}^{k=n-1} P(k)$$

... as equivalent to

$$\forall k \in \mathbb{N}, k < n \Rightarrow P(k)$$

More dominoes

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$$\left(\forall n \in \mathbb{N}, \left[\bigwedge_{k=0}^{k=n-1} P(k)\right] \Rightarrow P(n)\right) \Rightarrow \forall n \in \mathbb{N}, P(n)$$

If all the previous cases always imply the current case
then all cases are true

complete induction outline

inductive step: state inductive hypothesis $H(n)$

derive conclusion $C(n)$: show that $C(n)$ follows from $H(n)$, indicating where you use $H(n)$ and why that is valid

verify base case(s): verify that the claim is true for any cases not covered in the inductive step

Wait! isn't that the same outline as simple induction?

Yes, we just modify the inductive hypothesis, $H(n)$ so that it assumes the main claim for every natural number from the starting point up to $n-1$, and the conclusion, $C(n)$, is now the main claim for n.

watch the base cases, part 1

$$f(n) = \begin{cases} 1 & n \leq 1 \\ \left[f(\lfloor\sqrt{n}\rfloor)\right]^2 + 2f(\lfloor\sqrt{n}\rfloor) & n > 1 \end{cases}$$

Check a few cases, and make a conjecture

For all natural numbers $n > 1$, $f(n)$ is a multiple of 3?

use the complete induction outline

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zero pair free binary strings, zpfbs...

Denote by $zpfbs(n)$ the number of binary strings of length n that contain no pairs of adjacent zeros. What is $zpfbs(n)$ for the first few natural numbers n?

what is $zpfbs(n)$?

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what is $zpfbs(n)$?

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Every natural number greater than 1 has a prime factorization

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prime factorization: representation as product of 1 or more primes

Every natural number greater than 1 has a prime factorization

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After a certain natural number n, every postage can be made up by combining $3-$ and $5-$ cent stamps

what is the "certain natural number"?

After a certain natural number n, every postage can be made up by combining $3-$ and $5-$ cent stamps

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After a certain natural number n, every postage can be made up by combining $3-$ and $5-$ cent stamps

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notes...