A2: most of these released... but grader is carefully working through about 50 Q2b...
I think they will be worth the wait...
T2: RSN

CSC236 fall 2018

languages: definitions

...plus some regular expressions...

Danny Heap

heap@cs.toronto.edu / BA4270 (behind elevators)

http://www.teach.cs.toronto.edu/~csc236h/fall/

416-978-5899

Using Introduction to the Theory of Computation, Chapter 7

Outline

formal languages

regular expressions

NFSAs

notes

some definitions

\Sigma = unicode

atomic (can't break apart), bounds the necessary resources

alphabet: finite, non-empty set of symbols, e.g. $\{a, b\}$ or $\{0, 1, -1\}$. Conventionally denoted Σ.

could be infinitely many strings, but each has a finite length

string: finite (including empty) sequence of symbols over an alphabet: abba is a string over $\{a, b\}$. Convention: ε is the empty string, never an allowed symbol, Σ^* is set of all strings over Σ.

language: Subset of Σ^* for some alphabet Σ. Possibly empty, possibly infinite subset. E.g. $\{\}$, $\{aa, aaa, aaaa, \ldots\}$.

N.B.: $\{\} \neq \{\varepsilon\}$. |{}| = 0 != 1 = |{\varepsilon}|

Many problems can be reduced to languages: logical formulas, identifiers for compilation, natural language processing. Key question is recognition:

Given language L and string s, is $s \in L$?

is s accepted by the FSA that accepts L?

Languages may be described either by descriptive generators (for example, regular expressions) or procedurally (e.g. finite state automata)

more notation — string operations

string length: denoted $|s|$, is the number of symbols in s, e.g. $|bba| = 3$.

|\varepsilon| = 0

$s = t$: if and only if $|s| = |t|$, and $s_i = t_i$ for $0 \leq i < |s|$.

s^R: reversal of s is obtained by reversing symbols of s, e.g. $1011^R = 1101$.

most commonly

st or $s \circ t$: concatenation of s and t — all characters of s followed by all those of t, e.g. $bba \circ bb = bbabb$.

s^k: denotes s concatenated with itself k times. E.g., $ab^3 = ababab$, $101^0 = \varepsilon$.

Σ^n: all strings of length n over Σ, Σ^* denotes all strings over Σ.

\Sigma^0 = {\varepsilon}

language operations

$\overline{L}$: Complement of L, i.e. $\Sigma^* - L$. If L is language of strings over $\{0,1\}$ that start with 0, then $\overline{L}$ is the language of strings that begin with 1 plus the empty string.

$L \cup L'$: union = L' \cup L

$L \cap L'$: intersection = L' \cap L

$L - L'$: difference != L' - L

Rev(L): $= \{s^R : s \in L\}$ not necessarily equal to L

!= L'L

concatenation: LL' or $L \circ L' = \{rt | r \in L, t \in L'\}$. Special cases $L\{\varepsilon\} = L = \{\varepsilon\}L$, and $L\{\} = \{\} = \{\}L$.

more language operations

exponentiation: L^k is concatenation of L k times. Special case,

$$L^0 = \{\varepsilon\}, \text{ including } L = \{\} \quad (!)$$

analogous to 0^0 = 1 --- See Donald Knuth

{}^2 = {\varepsilon}{}{} = {}{}

\forall x \in \R, x != 0 => x^0 = 1, \forall x \in \R^+, 0^x = 0

Kleene star: $L^* = L^0 \cup L^1 \cup L^2 \cup \ldots$

{}* = {\varepsilon} = {\varepsilon}*

another way to define languages

In addition to the language accepted by DFSA $L(M)$ and set description $L = \{\ldots\}$.

regular expressions are themselves a language over --- what alphabet?
each string in the RE language denotes a language

Definition: The regular expressions (regexps or REs) over alphabet Σ is the **smallest** set such that:

e.g. if \Sigma = {a, b}, then basis \emptyset, \varepsilon, a, b

1. $\emptyset$, ε, and x, for every $x \in \Sigma$ are REs over Σ
2. if T and S are REs over Σ, then so are:
   - $(T + S)$ (union) — lowest precedence operator
   - (TS) (concatenation) — middle precedence operator
   - T^* (star) — highest precedence

regular expression to language:

The $L(R)$, the language denoted (or described) by R is defined by structural induction:

Basis: If R is a regular expression by the basis of the definition of regular expressions, then define $L(R)$:

- $L(\emptyset) = \emptyset$ (the empty language — no strings!)
- $L(\varepsilon) = \{\varepsilon\}$ (the language consisting of just the empty string)
- $L(x) = \{x\}$ (the language consisting of the one-symbol string)

Induction step: If R is a regular expression by the induction step of the definition, then define $L(R)$:

- $L(S + T) = L(S) \cup L(T)$
- $L(ST) = L(S)L(T)$
- $L(T^*) = L(T)^*$

regexp examples

- csc207 regex practice
- regex crosswords
- command-line REs
- $L(0+1) = \{0, 1\}$ = L(0) \cup L(1)
- $L((0+1)^*)$ All binary strings over $\{0, 1\}$ = L(0+1)*
- $L((01)^*) = \{\varepsilon, 01, 0101, 010101, \ldots\}$
- $L(0^*1^*)$ 0 or more 0s followed by 0 or more 1s.
- $L(0^* + 1^*)$ 0 or more 0s or 0 or more 1s.
- $L((0+1)(0+1)^*)$ Non-empty binary strings over $\{0, 1\}$.

example

$L = \{x \in \{0,1\}^* \mid x \text{ begins and ends with a different bit}\}$

L' = L((1(0+1)*0)+(0(0+1)*1))

to show that L = L', must show L \subseteq L' and L' \subseteq L

prove L' \subseteq L:
Let s \in L'. Then s \in L((1(0+1)*0)+(0(0+1)*1)) = L(1(0+1)*0) \cup L(0(0+1)*1). WLOG, assume s \in L(1(0+1)*0), since the same argument works for the other case by interchaning 0s and 1s.

Since s \in L(1(0+1)*0) = L(1)L(0+1)*L(0), s = tuv, where t = 1, v = 0, and u... well we don't care about u. So the first (and only) character of t (and hence s) is 1, and the last (and only) character of v (hence s) is 0. So s \in L.

prove L \subseteq L': Let s \in L. Then either s starts with 0, ends with 1, or starts with 1, ends with 0. WLOG assume s starts with 0, ends with 1. Then s = 0u1, where u \in L(0+1)*, so s \in L(0)L(0+1)*L(1) \subseteq L(0)L(0+1)*L(1) \cup L(1(0+1)*0) = L'

QED

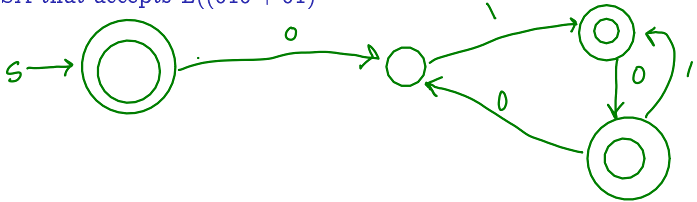
RE identities

some of these follow from set properties...
others require some proof (see 7.2.5 example)

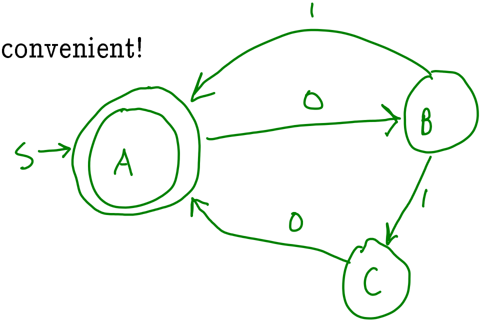
- communitativity of union: $R + S \equiv S + R$
- associativity of union: $(R + S) + T \equiv R + (S + T)$
- associativity of concatenation: $(RS)T \equiv R(ST)$
- left distributivity: $R(S + T) \equiv RS + RT$
- right distributivity: $(S + T)R \equiv SR + TR$
- identity for union: $R + \emptyset \equiv R$
- identity for concatenation: $R\varepsilon \equiv R \equiv \varepsilon R$
- annihilator for concatenation: $\emptyset R \equiv \emptyset \equiv R\emptyset$
- idempotence of Kleene star: $(R^*)^* \equiv R^*$

non-deterministic FSA (NFSA) example

FSA that accepts $L((010 + 01)^*)$



convenient!

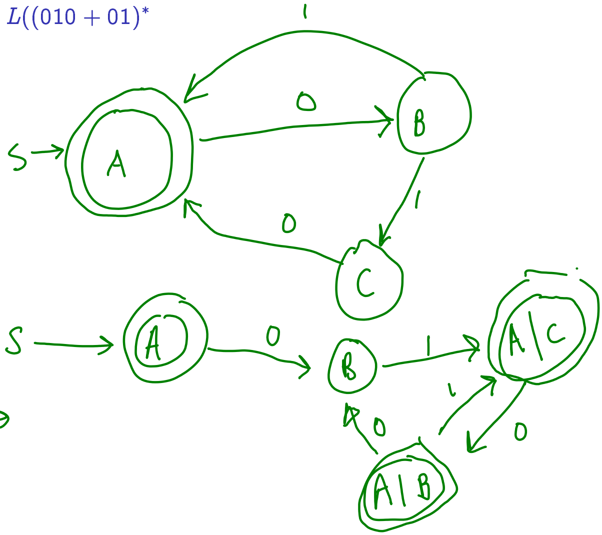


non-deterministic FSA (NFSA) example

FSA that accepts $L((010 + 01)^*)$

NFSA →

DFSA →



NFSAs are real

...you can always convert them to DFSA

Use subset construction, notes page 219

notes