CSC236 fall 2018

theory of computation

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use Introduction to the Theory of Computation, Section 1.2

Outline

introduction

chaper 1, simple induction

induction trap

Notes

why reason about computing?

- you're not just hackers anymore...
- can you test **everything**?
- careful, you might get to like it... (?!*)

how to reason about computing

- it's messy...

- it's art...

how to do well

- read the course information sheet as a two-way promise
- question, answer, record, synthesize
- collaborate with respect

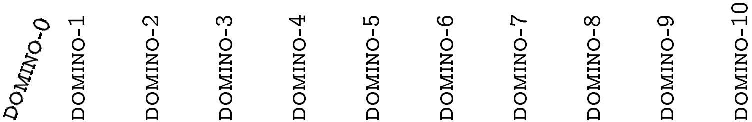
we behave as though you already know...

- Chapter 0 material from *Introduction to Theory of Computation*
- CSC165 material, especially proofs and big-Oh material
- But you can *relax* the structure a little (more on this later)
- recursion, efficiency material from CSC148

by December you'll know...

- understand, and use, several flavours of induction
- complexity and correctness of programs — both recursive and iterative
- formal languages, regular languages, regular expressions

domino fates foretold



$$[\,P(0) \;\wedge\; (\forall n \in \mathbb{N}, P(n) \Rightarrow P(n+1)\,)\,] \Longrightarrow \forall n \in \mathbb{N}, P(n)$$

If the initial case works,
and each case that works implies its successor works,
then all cases work

simple induction outline

inductive step: introduce n and inductive hypothesis $H(n)$

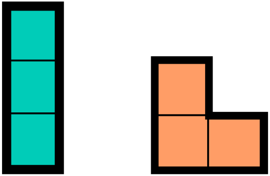
derive conclusion $C(n)$: show that $C(n)$ follows from $H(n)$, indicating **where** you use $H(n)$ and why that is valid

verify base case(s): verify that the claim is true for any cases not covered in the inductive step

in simple induction $C(n)$ is just $H(n+1)$

trominoes

see https://en.wikipedia.org/wiki/Tromino



Can an $n \times n$ square grid, **with one subsquare removed**, be tiled (covered without overlapping) by “chair” trominoes?

trominoes

Can an $n \times n$ square grid, **with one subsquare removed**, be tiled (covered without overlapping) by "chair" trominoes? Try small examples: $n = 1, 2, 3, 4, \ldots$

trominoes

$P(n)$ a $2^n \times 2^n$ square grid, **with one subsquare removed**, can be tiled (covered without overlapping) by "chair" trominoes?

base case: let's argue about where to start...

trominoes

$P(n)$ a $2^n \times 2^n$ square grid, **with one subsquare removed**, can be tiled (covered without overlapping) by "chair" trominoes?

inductive step:

$3^n \geq n^3$?

scratch work: check for a few values of n

$3^n \geq n^3$

use the simple induction outline

$3^n \geq n^3$

use the simple induction outline

For every $n \in \mathbb{N}, 12^n - 1$ is a multiple of 11

scratch work: substitute a few values for n

For every $n \in \mathbb{N}$, $12^n - 1$ is a multiple of 11

use the simple induction outline

For every $n \in \mathbb{N}, 12^n - 1$ is a multiple of 11

use the simple induction outline

The units digit of 7^n is one of 1, 3, 7, or 9

scratch work: substitute a few values for n

The units digit of 7^n is one of 1, 3, 7, or 9

use the simple induction outline

The units digit of 7^n is one of 1, 3, 7, or 9

use the simple induction outline

What about: the units digit of 7^n is one of 1, 2, 3, 7, or 9

use the simple induction outline

is the claim still true? What happens if you add this other case to the inductive step?

how **not** to do simple induction

does a graph $G = (V, E)$ with $|V| > 0$ have $|E| = |V| - 1$?

base case: easy

inductive step: what happens if you try to extend an arbitrary graph with $|V| = n$ to one with $|V'| = n + 1$?

variations: what happens if you restrict the claim to connected graphs? what about acyclic connected graphs?

take-home: decompose rather than construct... except in structural induction (later)

Notes