CSC236 tutorial exercises, Week #6

Here are your tutorial sections:

Surname	Time	Room	TA
A-K	Friday 11	SS1088	Zhaowei
L-Tg	Friday 11	SS2105	Hamed
Th-Z	Friday 11	BA2175	Gal
A-L	Friday noon	AB114	Wen
M-Z	Friday noon	BF323	Lauren
A-K	Friday 1	BA1170	Ammar
L-Tg	Friday 1	AB107	Alex
Th-Z	Friday 1	AB114	Shems
A-K	Thursday 8	BA2139	Zach
L-Tg	Thursday 8	BA2185	Ekansh
Th-Z	Thursday 8	BA2195	Danniel

These exercises are intended to give you practice with recurrence relations.

1. Consider the recurrence relation

$$T(n) = \begin{cases} 1 & n = 1 \\ 1 + T\left(\left\lceil \frac{n}{3} \right\rceil\right) & n > 1 \end{cases}$$

Use complete induction to prove that for every positive natural number $n, T(n) \geq c\ lg(n)$, for some positive real constant c.

2. Consider the recurrence relation

$$T(n) = \begin{cases} 1 & n = 0 \\ 3 & n = 1 \\ 3T(n-1) - 2T(n-2) & n > 1 \end{cases}$$

Find a closed form for $T(n)$, and prove that it is correct using induction.

3. Consider another recurrence relation

$$T(n) = \begin{cases} 1 & n = 0 \\ T(n-1) + n - 2 & n > 0 \end{cases}$$

Unwind the recurrence **carefully**, following the pattern below, for some n that is comfortably greater than 1:

$$\begin{aligned} T(n) &= T(n-1) + n - 2 \\ &= T(n-2) + n - 1 - 2 + n - 2 = T(n-2) + 2n - 5 \\ &= T(n-3) + n - 2 - 2 + 2n - 5 = T(n-3) + 3n - 9 \\ &\quad . \\ &\quad . \end{aligned}$$

Continue to see a pattern that leads to a guess at a closed form for $T(n)$.