CSC236 tutorial exercises, Week #12

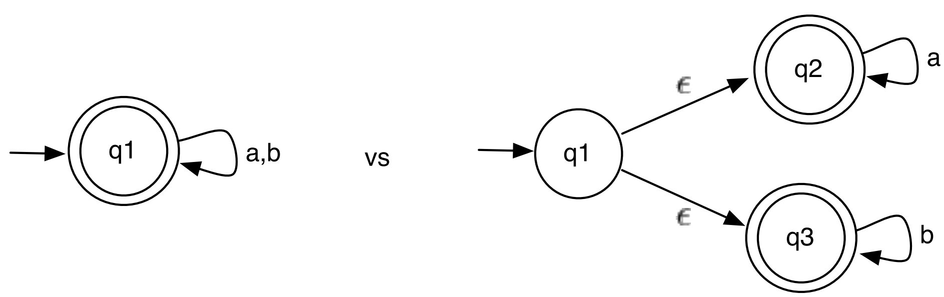
(best before Thursday afternoon)

Here are your tutorial sections:

Surname	Time	Room	TA
A–K	Friday 11	SS1088	Zhaowei
L–Tg	Friday 11	SS2105	Hamed
Th–Z	Friday 11	BA2175	Gal
A–L	Friday noon	AB114	Wen
M–Z	Friday noon	BF323	Lauren
A–K	Friday 1	BA1170	Ammar
L–Tg	Friday 1	AB107	Alex
Th–Z	Friday 1	AB114	Shems
A–K	Thursday 8	BA2139	Zach
L–Tg	Thursday 8	BA2185	Ekansh
Th–Z	Thursday 8	BA2195	Danniel

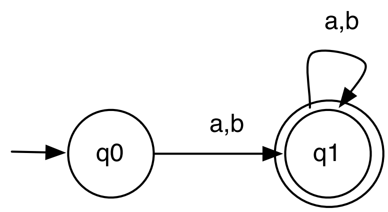
Let the alphabet be $\Sigma = \{a, b\}$

1. Are regular expressions $(a+b)^*$ and $a^* + b^*$ equivalent? Explain.
   Solution: Let $R_1 = (a+b)^*$ and $R_2 = a^* + b^*$. $R1 \neq R2$, because R_1 includes all strings in the alphabet Σ while R_2 includes repetitions of a (including 0 repetitions) or repetitions of b, but no strings that include both a and b. The corresponding NFSA are different as well:



2. Draw a DFSA corresponding to the regular expression $(a+b)(a+b)^*(a^*+b^*)$. Write down the corresponding state invariant that you could use to prove the equality of your DFSA to the regular language represented by the provided regexp. You don't need to provide the proof.
   Solution: First, note that $(a^* + b^*) \subset (a+b)^*$ and not only that, but any string that can be generated

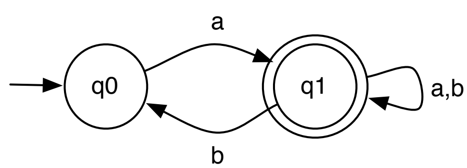
by $(a+b)^*(a^*+b^*)$ can be generated by $(a+b)^*$ due to the definition of the Kleene's star. Hence, we can simplify $(a+b)(a+b)^*(a^*+b^*) = (a+b)(a+b)^*$. The corresponding DFSA is



State invariants are then as follows:

$$\delta^*(q_0, s) = \begin{cases} q_1 & \texttt{if s starts with an } a \texttt{ or a } b \\ q_0 & \texttt{otherwise} \end{cases}$$

3. Consider an FSA M_1:

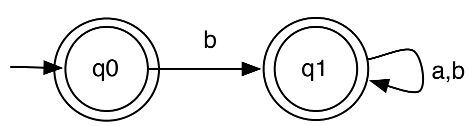


(a) Is this a DFSA or an NFSA? Why?
Answer: This is an NFSA, because $\delta(q_1, b) = \{q_1, q_0\}$, i.e. there is more than one path to take upon observing b in state q_1. Also, the transition $\delta(q_0, b)$ is not determined but sometimes dead states are omitted even in drawing DFSA, so you have to be careful calling FSA an NFSA just because dead states are missing.

(b) Write down the language $\mathcal{L}$ that it represents (a sentence describing all strings included in the language $\mathcal{L}$)
Answer: This language contains all strings that start with an a

(c) Write down the complement $\overline{\mathcal{L}}$ of $\mathcal{L}$, i.e. $\overline{\mathcal{L}} = \Sigma^* - \mathcal{L}$ in one sentence
Answer: Language $\overline{\mathcal{L}}$ contains all strings in Σ^* that do not start with an a (including empty string).

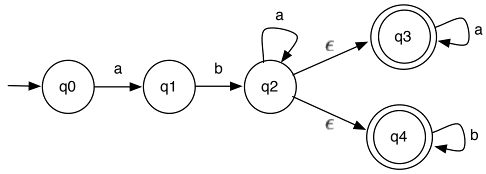
(d) Draw an FSA for $\overline{\mathcal{L}}$



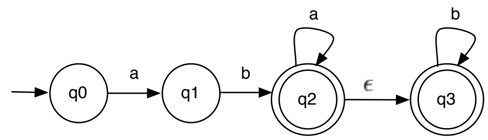
4. Consider a regexp R_1: $a(ba^*)(a^*+b^*)$

(a) Draw an NFSA M_2 corresponding to the R_1 above

Solution: There are many answers to this question, including



and



The second solution follows from $a(ba^*)(a^* + b^*) = ab(a^*a^* + a^*b^*) = ab(a^* + a^*b^*) = aba^*b^*$

(b) Write down the language $\mathcal{L}$ that it represents (a sentence describing all strings)
Answer: $\mathcal{L}$ contains all strings that start with ab, concatenated with repetitions of a (including zero repetitions of a) followed by repetitions of b (including zero repetitions of b).

(c) Draw a corresponding DFSA