CSC236 tutorial exercises, Week #9
Sample Solutions

1. **Note that we want to show $S(n)$ <u>for all $n \geq B \ \in \mathbb{N}$</u> is in $O(\lg \lg n)$.**

   Sample Solution 1.

   Assume $\sqrt{2^{2^k}} \leq n \leq 2^{2^k}$

   $S(n) \leq S\left(2^{2^k}\right)$ # since S is non-decreasing

   $S(n) \leq \lg \lg 2^{2^k} + 3$

   $S(n) \leq \lg \lg n^2 + 3$ # since $\sqrt{2^{2^k}} \leq n$, then $2^{2^k} \leq n^2$

   $S(n) \leq \lg(2 \lg n) + 3$

   $S(n) \leq 1 + \lg \lg n + 3$

   $S(n) \leq \lg \lg n + 4$

   $S(n) \leq \lg \lg n + 4 \lg \lg n$ # when $n \geq 4$

   $S(n) \leq 5 \lg \lg n$

   C= 5 and B=4 □

   Sample Solution 2.

   Assume $\sqrt{2^{2^k}} \leq n \leq 2^{2^k}$

   $S(n) \leq S\left(2^{2^k}\right)$ # since S is non-decreasing

   $S(n) \leq \lg \lg 2^{2^k} + 3$

   $S(n) \leq k + 3$

   $S(n) \leq (k - 1) + 4$

   $S(n) \leq \lg \lg 2^{(2^{k-1})} + 4$

   $S(n) \leq \lg \lg \sqrt{2^{2^k}} + 4$

   $S(n) \leq \lg \lg n + 4$ # since lg is monotonic

   $S(n) \leq \lg \lg n + 4 \lg \lg n$ # when $n \geq 4$

   $S(n) \leq 5 \lg \lg n$

   C= 5 and B=4 □

2. **Note that we want to show $S(n)$ <u>for all $n \geq B \in \mathbb{N}$</u> is in $O(\lg \lg n)$.**

Sample Solution 1.

Assume $\sqrt{2^{2^k}} \leq n \leq 2^{2^k}$

$S(n) \leq S\left(2^{2^k}\right)$ # since S is non-decreasing

$S(n) \leq \lg \lg 2^{2^k} + 4$

$S(n) \leq \lg \lg n^2 + 4$ # since $\sqrt{2^{2^k}} \leq n$, then $2^{2^k} \leq n^2$

$S(n) \leq \lg (2 \lg n) + 4$

$S(n) \leq 1 + \lg \lg n + 4$

$S(n) \leq \lg \lg n + 5$

$S(n) \leq \lg \lg n + 5 \lg \lg n$ # when $n \geq 4$

$S(n) \leq 6 \lg \lg n$

C= 6 and B=4

$\square$

Sample Solution 2.

Assume $\sqrt{2^{2^k}} \leq n \leq 2^{2^k}$

$S(n) \leq S\left(2^{2^k}\right)$ # since S is non-decreasing

$S(n) \leq \lg \lg 2^{2^k} + 4$

$S(n) \leq k + 4$

$S(n) \leq (k - 1) + 5$

$S(n) \leq \lg \lg 2^{(2^{k-1})} + 5$

$S(n) \leq \lg \lg \sqrt{2^{2^k}} + 5$

$S(n) \leq \lg \lg n + 5$ # since lg is monotonic

$S(n) \leq \lg \lg n + 5 \lg \lg n$ # when $n \geq 4$

$S(n) \leq 6 \lg \lg n$

C= 6 and B=4

$\square$

3. **Note that we want to show $S(n)$ <u>for all $n > 1 \in \mathbb{N}$</u> is in $\Omega(\lg \lg n)$.**

Sample Solution.

Assume $\sqrt{2^{2^k}} \leq n \leq 2^{2^k}$

$S(n) \geq S\left(\sqrt{2^{2^k}}\right)$ # since S is non-decreasing

$S(n) \geq \lg \lg \sqrt{2^{2^k}} + 3$

$S(n) \geq \lg \lg 2^{(2^{k-1})} + 3$

$S(n) \geq k - 1 + 3$

$S(n) \geq k$

$S(n) \geq \lg \lg n$ # since $k = \lg \lg n$

$\square$