CSC236 tutorial exercises, Week #10

(best before Thursday afternoon)

Here are your tutorial sections:

Surname	Time	Room	TA
A–K	Friday 11	SS1088	Zhaowei
L–Tg	Friday 11	SS2105	Hamed
Th–Z	Friday 11	BA2175	Gal
A–L	Friday noon	AB114	Wen
M–Z	Friday noon	BF323	Lauren
A–K	Friday 1	BA1170	Ammar
L–Tg	Friday 1	AB107	Alex
Th–Z	Friday 1	AB114	Shems
A–K	Thursday 8	BA2139	Zach
L–Tg	Thursday 8	BA2185	Ekansh
Th–Z	Thursday 8	BA2195	Danniel

Find the complexity class for functions below WITHOUT using induction. Note that the function $S(n)$ is given ONLY for special cases of n. Assume $S(n)$ is monotonic non-decreasing for all n.

1. Assume we know that when $n = 2^{(2^k)}$ for some $k \in \mathbb{N}$, $S(n) = \lg\lg n + 3$. Show that $S(n) \in O(\lg\lg n)$ for all $n \geq B, n \in \mathbb{N}$ (not just special cases of n as defined above) and find a value for B.
2. Assume we know that when $n = 2^{(2^k)}$ for some $k \in \mathbb{N}$, $S(n) = \lg\lg n + 4$. Show that $S(n) \in O(\lg\lg n)$ for all $n \geq B, n \in \mathbb{N}$ (not just special cases of n as defined above) and find a value for B.
3. Assume we know that when $n = 2^{(2^k)}$ for some $k \in \mathbb{N}$, $S(n) = \lg\lg n + 3$. Show that $S(n) \in \Omega(\lg\lg n)$ for all $n > 1, n \in \mathbb{N}$ (not just special cases of n as defined above).

Hints: For the three problems above, you may use the following truths:

- **i.** You may use monotonicity and bounds to find the complexity class.
- **ii.** $\forall n > 1, n \in \mathbb{N}, \exists k \in \mathbb{N}$ such that $\sqrt{2^{(2^k)}} \leq n \leq 2^{(2^k)}$
- **iii.** $\sqrt{2^{(2^k)}} = 2^{(2^{k-1})}$
- **iv.** For $n = 2^{(2^k)}, 2^k = \lg n$ and $k = \lg\lg n$