

CSC236 fall 2016

complete induction

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use Introduction to the Theory of Computation,
Section 1.3



Complete Induction

another flavour needed

Every natural number greater than 1 has a prime factorization

Try some examples

How does the factorization of 8 help with the factorization of 9?

More dominoes



$$(\forall n \in \mathbb{N}, \langle P(0), \dots, P(n-1) \rangle \Rightarrow P(n)) \Rightarrow \forall n \in \mathbb{N}, P(n)$$

If all the previous cases always implies the current case
then all cases are true

complete induction outline

inductive step: state inductive hypothesis $H(n)$

derive conclusion $C(n)$: show that $C(n)$ follows from $H(n)$, indicating where you use $H(n)$ and why that is valid

verify base case(s): verify that the claim is true for any cases not covered in the inductive step

Wait! isn't that the same outline as simple induction?

Yes, we just modify the inductive hypothesis, $H(n)$ so that it assumes the main claim for every natural number from the starting point up to $n - 1$, and the conclusion, $C(n)$, is now the main claim for n .

watch the base cases, part 1

$$f(n) = \begin{cases} 1 & n \leq 1 \\ [f(\lfloor \sqrt{n} \rfloor)]^2 + 2f(\lfloor \sqrt{n} \rfloor) & n > 1 \end{cases}$$

Check a few cases, and make a conjecture



For all natural numbers $n > 1$, $f(n)$ is a multiple of 3?
use the complete induction outline



For all natural numbers $n > 1$, $f(n)$ is a multiple of 3?
use the complete induction outline



zero pair free binary strings, zpfbs...

Denote by $zpfbs(n)$ the number of binary strings of length n that contain no pairs of adjacent zeros. What is $zpfbs(n)$ for the first few natural numbers n ?

what is $zpfbs(n)$?

use the complete induction outline



what is $zpfbs(n)$?

use the complete induction outline



After a certain natural number n , every postage can be made up by combining 3– and 5– cent stamps

what is the “certain natural number”?



After a certain natural number n , every postage can be made up by combining 3– and 5– cent stamps

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After a certain natural number n , every postage can be made up by combining 3– and 5– cent stamps

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Every natural number greater than 1 has a prime factorization

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prime factorization: representation as product of 1 or more primes



Every natural number greater than 1 has a prime factorization

use the complete induction outline

