

In the office hours this week, for the most part, we discussed/reviewed the `closestPair` divide and conquer algorithm. Some major points we discussed is as follows:

- The problem in `closestPairDCV0` is that after dividing the list of n points to two halves, the conquer part in which the closest points around the partitioning line is calculated is in $O(n^2)$, as it's basically carried out in two nested loops:

```
for ... n
  for ... n
```

This, by the master theorem, makes the overall time complexity of the algorithm in $O(n^2 \log n)$.

- In `closestPairDCV1`, we benefit from the fact that there is no need to calculate the distance of every point near the partitioning line to every other points in that region. Instead, all it's needed is to calculate the distance of each of those points to a fixed amount (for example 5, or even 3) of other points in that region provided that those points are sorted based on their y-coordinates. Hence, the complexity of the conquer part is $O(n \log n)$, as it's basically carried out as follows:

```
mergeSort(list, on y-coordinates)
for ... n
  for ... fixed c (e.g., 5)
```

} $O(n \log n)$

This, by unwinding, makes the overall time complexity of the algorithm in $O(n \log^2 n)$.

- Finally in `closestPairDC`, we benefit from the fact that as the original list of points is being partitioned (divided to two halves) in a merge sort manner, we should not call `mergeSort` from scratch. Instead, we can just call the merge function. Hence, the complexity of the conquer part is $O(n)$, as it's basically carried out as follows:

```
merge(list, on y-coordinates)
for ... n
  for ... fixed c (e.g., 5)
```

} $O(n)$

This, by the master theorem, makes the overall time complexity of the algorithm in $O(n \log n)$.

- We also discussed, to some extent, Questions 2 to 4 of the assignment 2:
 - Q2 is on Full Binary Trees, not Binary Trees. Also, in Q2, nodes do not have labels; hence, the distinctness is about the tree structure (shape).
 - There cannot be more than one interpretation for Q3: taking into account 1) the problem context (microorganisms) 2) odd defined are undefined, and 3) to prove T is strictly increasing.

- In Q4, the order does not count. “most even numbers” means your recurrence relation does not have to be correct for small even numbers; but after certain points, it should be correct for all other even numbers.
- We also discussed common mistakes of Test 1 as follows:
 - Putting quantification inside definition of $P(n)$ in proof by induction.
 - Putting quantification inside inductive hypothesis in proof by induction.
 - Not assuming arbitrary elements in the inductive hypothesis of structural induction.
- Some students asked about [remark procedure](#) and the deadline. The deadline is 7 days after the work is returned. You should feel free to discuss your Test and/or Assignment with me at any point.
- Some students asked if they can discuss their Test 1 and Assignment 1 in Peer Instruction Sessions; and, the answer is certainly “yes”. Here, it’s the updated list of [mentors](#).