

In the office hour this week, we mostly reviewed Example 61. In particular, we discussed different variants of the recurrence relation of the *binSearch*,

$$T(n) = \begin{cases} c_1 & n = 1 \\ c_2 + \text{either } T(m - b + 1) \text{ or } T(e - m) & n > 1 \end{cases}$$

$$T(n) \cong \begin{cases} c_1 & n = 1 \\ c_2 + T\left(\frac{n}{2}\right) & n > 1 \end{cases}$$

$$T(n) = \begin{cases} c_1 & n = 1 \\ c_2 + \text{either } T\left(\left\lceil \frac{n}{2} \right\rceil\right) \text{ or } T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) & n > 1 \end{cases}$$

$$T(n) = \begin{cases} c_1 & n = 1 \\ c_2 + T\left(\left\lceil \frac{n}{2} \right\rceil\right) & n > 1 \end{cases} \quad \text{worst case}$$

$$T(n) = \begin{cases} 1 & n = 1 \\ 1 + \text{either } T\left(\left\lceil \frac{n}{2} \right\rceil\right) \text{ or } T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) & n > 1 \end{cases}$$

$$T(n) = \begin{cases} 1 & n = 1 \\ 1 + T\left(\left\lceil \frac{n}{2} \right\rceil\right) & n > 1 \end{cases} \quad \text{worst case}$$

We also saw both intuitively and mathematically why $m - b + 1 = \left\lceil \frac{n}{2} \right\rceil$

$$\text{Since } m = \left\lfloor \frac{e+b}{2} \right\rfloor$$

$$\begin{aligned} m - b + 1 &= \left\lfloor \frac{e+b}{2} \right\rfloor - b + 1 \\ &= \left\lfloor \frac{e+b}{2} - b + 1 \right\rfloor = \left\lfloor \frac{e-b+1}{2} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{n}{2} + \frac{1}{2} \right\rfloor = \left\lceil \frac{n}{2} \right\rceil \end{aligned}$$

We left as a practice for you to show $e - m = \left\lfloor \frac{n}{2} \right\rfloor$