

## CSC236 Intro. to the Theory of Computation

### Lecture 5: Recurrences and D&C

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Course page:  
<http://www.cdf.toronto.edu/~csc236h/fall/index.html>

Section page:  
[http://www.cdf.toronto.edu/~csc236h/fall/amir\\_lectures.html](http://www.cdf.toronto.edu/~csc236h/fall/amir_lectures.html)

Recurrences 5-1

## review

- ❖ **so far**
  - different variants of induction
  - recurrence relations
  - introduced the application of recurrence relations to complexity of recursive algorithms
- ❖ **this week**
  - application of recurrence relations to complexity of Divide & Conquer algorithms

Recurrences 5-2

## recursive algorithms

- ❖ **normally reduce/split the problem to some problems of smaller size**
  - $\text{factorial}(n-1)$  is smaller vs.  $\text{factorial}(n)$
  - $\text{fib}(n-1)$  and  $\text{fib}(n-2)$  are smaller vs.  $\text{fib}(n)$
  - $\text{mergeSort}(A, 1^{\text{st}} \text{ half})$  and  $\text{mergeSort}(A, 2^{\text{nd}} \text{ half})$  are smaller vs.  $\text{mergeSort}(A)$
  - $\text{binSearch}(x, A, 1^{\text{st}} \text{ half})$  and  $\text{binSearch}(x, A, 2^{\text{nd}} \text{ half})$  are smaller vs.  $\text{binSearch}(x, A)$
- ❖ **recurrences**
  - towards the complexity of D&C Alg.

Recurrences 5-3

## Example 61: binSearch

```
def binSearch(x, A, b, e):  
    if b == e:  
        if x == A[b]:  
            return b  
        else:  
            return -1  
    else:  
        m = (b + e) // 2          # midpoint  
        if x <= A[m]:  
            return binSearch(x, A, b, m)  
        else:  
            return binSearch(x, A, m+1, e)
```

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## Example 61: binSearch

- ❖ **a recurrence relation for complexity of binSearch**

Recurrences and D&C 5-5

## Example 61: binSearch

- ❖ **guessing (roughly calculating) a closed form**

Recurrences and D&C 5-6

### Example 61: binSearch

- ❖ calculating a lower bound

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### Example 61: binSearch

- ❖ calculating a lower bound

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### Example 61: binSearch

- ❖ calculating an upper bound

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### Example 61: binSearch

- ❖ calculating an upper bound

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notes:

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notes:

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