

Let examples 55 and 56 be the two questions that you have in [Tutorial 3](#) on structural induction. Following examples are on recurrences, using induction to prove properties on recurrences, and finding closed forms of recurrences.

- **Example 57.** This week, we calculated the closed form of Fibonacci as follows.

$$Fib(n) = \frac{1}{\sqrt{5}} \cdot \left(\frac{1 + \sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}} \cdot \left(\frac{1 - \sqrt{5}}{2}\right)^n$$

Use induction, to prove what we have found is actually correct.

- **Example 58.** Without using the closed form of Fibonacci, prove $\sum_{j=0}^m Fib(j) < Fib(m+2)$.
- **Example 59.** Assume you have exactly \$1 today. Also, assume your money will be doubled every day and 99% of whatever you have each day will be lost after two days. How much money will you have after 5 months? Guess it. Then, calculate it.
- **Example 60.** Find the recurrence relation, $T(n)$, for number of *moves* in the Tower of Hanoi? Then, find its closed form. The theorem we discussed in Example 52 does not help here. **Hint:** Unwind $T(n)$ by substituting it into itself: replace $T(n-1)$ in terms of $T(n-2)$, ..., all the way to $T(1)$. This technique should lead you to find the closed form.

Note. We do not intend to publish solutions (or solutions outline) for any of the questions of the course notes, or extra practices. You are more than welcome to discuss your solutions with us.

Always, do some scratch work first to verify if the claim makes sense or not. If it does, prove it. If it does not, you probably have a counter example already.