CSC236 Fall 2016

Office Hour 04

In the office hour this week, in addition to quick clarifications on questions of Assignments 1, we discussed detailed proof of Example 54 as well as similarities/differences between simple and strong inductions.

- Example 54. We mostly discussed the strong induction proof, with some insights to its simple induction variant. Here, I present a proof by simple induction:

  Let $P(n)$ denote $f(n) < 2^{n+2}$ where $f(n) = \begin{cases} 2 & n = 0 \\ 7 & n = 1 \\ 2f(n-2) + f(n-1) & n > 1 \end{cases}$

  Proof by strong induction.

  Basis step. $P(0)$ and $P(1)$ hold as $2 < 2^{0+2}=4$ and $7 < 2^{1+2}=8$, respectively.

  Inductive step. Assume $P(i)$ holds for $2 \leq i \leq k$ and arbitrary fixed $k \geq 3 \in \mathbb{N}$; i.e.,

  $f(i) < 2^{i+2}$.

  We must show $P(k+1)$ holds too, i.e., $f(k+1) < 2^{k+3}$

  $f(k+1) = 2f(k-1) + f(k)$ by definition of f

  $< 2.\ 2^{k+1} + 2^{k+2}$ by IH, since $3 \leq k$, $2 \leq k\text{-}1 \leq k$

  $< 2^{k+2} + 2^{k+2}$

  $< 2^{k+3}$

  This completes the inductive step. Hence. $f(n) < 2^{n+2}$ $\forall n \in \mathbb{N}$.

□