

CSC236 Intro. to the Theory of Computation

Lecture 4: Recurrences

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Course page:
<http://www.cdf.toronto.edu/~csc236h/fall/index.html>

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http://www.cdf.toronto.edu/~csc236h/fall/amir_lectures.html

Recurrences 4-1

review

- ❖ **So far**
 - Simple Induction, Strong Induction, WOP, and Structural Induction
- ❖ **over 48 examples**
- ❖ **This week**
 - Recurrence relations, closed forms, and
 - proof of their properties

Recurrences 4-2

Example 50: rabbits

- ❖ A rabbit couple lives in an island. They are newborn and do not breed until they are 2 months old. Since age 2-month, each couple produces another couple per month. Find a recurrence relation for the number of couples after n months, assuming they never die.

Structural Induction 4-3

Example 50:

Structural Induction 4-4

Example 51: "00" free strings

- ❖ Find a recurrence relation for the number of binary strings of length n that do not contain substring "00". (Revisit of Example 27.)

Structural Induction 4-5

Example 52: finding closed form

- ❖ Assume $f_n = c_1 f_{n-1} + c_2 f_{n-2}$
 - Find roots of $r^2 - c_1 r - c_2 = 0$, r_1 and r_2 .
 - Then solve $f_n = \alpha_1 r_1^n + \alpha_2 r_2^n$, using the initial values of f .

Structural Induction 4-6

Example 52:

Structural Induction 4-7

Example 53:

$$f(n) = \begin{cases} 2 & n = 0 \\ 7 & n = 1 \\ 2f(n-2) + f(n-1) & n > 1 \end{cases}$$

Find the closed form of f .

Structural Induction 4-8

Example 53:

Structural Induction 4-9

Example 54:

Prove $f(n) < 2^{n+2}$. $f(n)$ defined in Examples 53.

Structural Induction 4-10

Example 54:

Structural Induction 4-11

notes:

Structural Induction 4-12