

Let examples 42 and 43 be the two questions that you have in [Tutorial 2](#) regarding complete induction. Here, there are more examples on **wop** and **structural induction**.

- **Example 44:** use wop to prove that there is no solution to
$$x^3 + 2y^3 + 4z^3 = 0$$
 in the set of natural numbers greater than zero.
- **Example 45:** use wop to prove that $\sqrt{2}$ is irrational.
- **Example 46:** let $\Sigma = \{0,1\}$. Assume Σ^* is defined as follows: $1 \in \Sigma^*$ and if $w \in \Sigma^*$ and $x \in \Sigma$, then $1xwx1 \in \Sigma^*$. Prove that number of 0's is less than number of 1's in all $w \in \Sigma^*$.
- **Example 47:** let $\Sigma = \{0,1\}$. Provide a recursive definition of binary palindrome strings of odd length, call it Σ^* . Make a conjecture. Prove it.
- **Example 48:** let $\Sigma = \{0,1\}$. Assume Σ^* is defined as follows: $0,1 \in \Sigma^*$ and if $w_1, w_2 \in \Sigma^*$, then $w_1 w_2 w_2 w_1 \in \Sigma^*$. Prove that $\forall w_i, w_j \in \Sigma^*$, the difference of $L(w_i)$ and $L(w_j)$ is a multiple of 3. $L(w)$ denotes length of w .

Note. We do not intend to publish solutions (or solutions outline) for any of the questions of the course notes, or extra practices. You are more than welcome to discuss your solutions with us.

Always, do some scratch work first to verify if the claim makes sense or not. If it does, prove it. If it does not, you probably have a counter example already.