CSC236 Fall 2016

Office Hour 02

In the office hour this week, we further discussed the following topics/examples:

- Constructive proofs. A constructive proof looks more like an algorithmic approach: we start from P(*k*)—or even from P(*b*)—and construct P(*k*+1) in a systematic way. (This might not be clear first, but starts to emerge with more scratch work.) Depending on what we use (only P(*k*) or some/all P(*i*)'s) to construct P(*k*+1), the proof falls into the simple or strong induction. Equivalently, we can start from P(*k*+1) and reduce it to only P(*k*) or some/all P(*i*)'s.
    - We discussed constructive approaches for some conjectures similar to the one in Example 22 (Chocolate Bar).
- Example 23: In class, we went over a strong induction approach that had one basis case; P(2); and in the Inductive Step, we had two cases:
    - Case 1: $\lfloor\sqrt{k+1}\rfloor = \lfloor\sqrt{k}\rfloor$
    - Case 2: $\lfloor\sqrt{k+1}\rfloor = \lfloor\sqrt{k}\rfloor + 1$

  Based on the fact (derived from definition of flor function) that for any two real numbers, when $r_2 = r_1 + \varepsilon$, then $\lfloor\sqrt{r_2}\rfloor$ is either $\lfloor\sqrt{r_1}\rfloor$ or $\lfloor\sqrt{r_1}\rfloor$+1.

  Then we continued the proof by showing that *f*(*k*+1) can be reduced to some of the *f*(*i*)'s in the inductive hypothesis for any *k*≥2.

  During the office hours, we discussed a slightly different proof (still by strong induction) in which we have two basis steps: P(2) and P(3); and in the Inductive Step, we have only one case in which we show that $\lfloor\sqrt{k+1}\rfloor$ can be reduced to some of the *f*(*i*)'s in the inductive hypothesis for any *k*≥3.

- Example 24:
    - We briefly discussed that Example 24 (polygon) is very similar to Example 22. We also discussed some hints for a constructive proof for Example 24 as well as some discussions towards making new conjectures similar to Example 22 and prove it constructively.
- Example 12': we also discussed why $n^3 \le 3^n$ $\forall n \ge 0$ cannot be proved just by induction, instead by exhaustive proof (AKA direct verification) for n∈{0,1,2} and by induction for *n*≥3
- Example 12'…': we went over two hints here: **1)** for what values of *n* we can prove $n^m \le m^n$ by simple inductions? Or, if we want to prove it for all *n*, we may want to restrict *m*, such as *m*≥3. **2)** as well as two different ways of approaching it; one by using the binomial theorem and continue the proof similar to Examples 12, 12', 12''; the other by using some math identity, such as $(k+1)^m=(1+1/k).k^m$

- We also emphasized that every proof by induction has at least one basis step even though the basis step may seem implicit in some approaches.