CSC236 Fall 2016

More Practice 01

This week we had 4 proof examples during lecture01 (Examples 1 to 4). We also saw an invalid proof (Example 5) for n non-parallel lines meeting in a common point, as well as another invalid proof for $2n \leq n^2+1 \ \forall n \in \mathbb{N}$ (let's call it Example 7). In Example 7, although the claim is true, the proof we discussed was wrong. The lesson in this example was to double check the starting point (b) is still correct by end of our proof. In other words, if during the Inductive Step, any constraints for starting point emerges, we should adjust the starting point. Prof Heap suggests a nice strategy: he starts off the proof from the Inductive Step (instead of the Basis Step). Please see his slides here,

In Lab01, you are going to see 3 more examples (let's call them Examples 8-10). Here are more examples for you to master your skills even further.

- **Example 11:** For which values of n, $2^n \geq 200n$ holds? Prove it by simple induction.
- **Example 12:** For which values of n, $n^2 \leq 2^n$ holds? Prove it by simple induction.
    - Examples 12': For which values of n, $n^3 \leq 3^n$ holds? Prove it by simple induction.
    - Examples 12''. For which values of n, $n^4 \leq 4^n$ holds? Prove it by simple induction.
    - Examples 12'$^{\cdots}$'. Prove $n^m \leq m^n$ holds for any arbitrary fixed $m \geq 2 \in \mathbb{N}$.
- **Example 13:** Prove or disprove $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n} \quad \forall n \geq 2 \in \mathbb{N}$.
- **Example 14:** Prove or disprove $11 \mid 12^n - 1 \ \ \forall n \in \mathbb{N}$.
- **Example 15:** Prove or disprove $2^n \leq n!$.
- **Example 16:** Let $|S|=n$, how many odd-sized subsets does S have? Prove your answer by simple induction, i.e. $(P(b) \wedge \forall k \geq b \in \mathbb{N}\ (P(k) \rightarrow P(k+1))) \rightarrow \ \forall n \geq b \in \mathbb{N}\ P(n)$.
- **Example 17:** Let $|S|=n$, how many subsets of cardinality 2 does S have? Prove your answer by simple induction, i.e. $(P(b) \wedge \forall k \geq b \in \mathbb{N}\ (P(k) \rightarrow P(k+1))) \rightarrow \ \forall n \geq b \in \mathbb{N}\ P(n)$.

Always, do some scratch work first to verify if the claim makes sense or not. If it does, prove it. If it does not, you probably have a counter example already.