CSC236 tutorial exercises #4

(Best before 11 am, Monday October 22nd)

Danny Heap

Here are your tutorial sections:

Surname	Section	Room	TA
A–F	Day 1 (11:00 am)	LM162	Yuval
G–Li	Day 2 (11:00 am)	BA2139	Lila
Lo–Si	Day 3 (11:00 am)	BA2145	Oles
So–Z	Day 4 (11:00 am)	BA2155	Lalla
A–H	Evening 1 (8:00 pm)	BA1190	Colin
I–M	Evening 2 (8:00 pm)	BA2135	Norman
N–Z	Evening 3 (8:00 pm)	BA2139	Feyyaz

These exercises are meant to give you practice with some of the concepts used to prove the Master Theorem.

1. Consider the recurrence:

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(\lceil n/2 \rceil) + T(\lfloor n/2 \rfloor) + n + 1 & \text{if } n > 1 \end{cases}$$

   This recurrence is superficially different from the one derived in the Course notes. Use the above recurrence and the approach of Lemma 3.6 in the Course Notes to show that T is non-decreasing.

2. Use repeated substitution (unwinding) to find a closed form for the recurrence S when n is a power of 3:

$$S(n) = \begin{cases} 1 & \text{if } n < 3 \\ a_1 S(\lceil n/3 \rceil) + a_2 S(\lfloor n/3 \rfloor) + n^2 & \text{if } n > 2 \end{cases}$$

   . . . where integers $a_1, a_2 \geq 0$ and $a_1 + a_2 = 3$.