1. Write detailed proof *structures* for each of the following statements. **Don't write complete proofs** — for now, focus on the proof structure only and leave out *all* of the actual "content".

   (a) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x \leqslant y \Rightarrow \exists z \in \mathbb{Z}, x \leqslant z \leqslant y$

   (b) $\forall x \in \mathbb{Z}, (\exists y \in \mathbb{Z}, x = 3y + 1) \Rightarrow (\exists y \in \mathbb{Z}, x^2 = 3y + 1)$

   (c) $\neg\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, y > x \wedge a_y > a_x$

2. Now, complete the proofs of each statement from the previous question.

   (a) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x \leqslant y \Rightarrow \exists z \in \mathbb{Z}, x \leqslant z \leqslant y$

   (b) $\forall x \in \mathbb{Z}, (\exists y \in \mathbb{Z}, x = 3y + 1) \Rightarrow (\exists y \in \mathbb{Z}, x^2 = 3y + 1)$

   (c) $\neg\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, y > x \wedge a_y > a_x$ — for the sequence $A = 2, 4, 6, 8, 9, 7, 5, 3, 1, 0, 0, 0, 0, 0, \ldots$