

To prove: $\forall x \in \mathbb{R}, \lfloor x \rfloor > x - 1$

Assume $x \in \mathbb{R}$ # in order to introduce $\forall$

Then $\lfloor x \rfloor \in \mathbb{Z}$ # definition of $\lfloor x \rfloor$

Then $\lfloor x \rfloor + 1 \in \mathbb{Z}$ # $1, \lfloor x \rfloor \in \mathbb{Z}$ & $\mathbb{Z}$ closed under $+$

Then $\lfloor x \rfloor + 1 > \lfloor x \rfloor$ # add $\lfloor x \rfloor$ to $1 > 0$

Then $\lfloor x \rfloor + 1 > x$ # contrapositive of

$\forall z \in \mathbb{Z}, z \leq x \Rightarrow z \leq \lfloor x \rfloor$

in defn of $\lfloor x \rfloor$

Then $\lfloor x \rfloor > x - 1$ # subtract 1 both sides.

Then $\forall x \in \mathbb{R}, \lfloor x \rfloor > x - 1$ # introduced $\forall$.