average...

$$\frac{\sum_{i \in \mathcal{I}_{f,n}} RT(i)}{|\mathcal{I}_{f,n}|}$$ ← = ∞

$$\mathcal{I}_{f,n} = \{i \mid i \text{ is an input to } f \wedge |i| = n\}$$

if list of ints →

$n = 4$, 2^4 inputs

```
def has_even(number_list):
    for number in number_list:
        if number % 2 == 0:
            return True
    return False
```

$1 \cdot 2^3$

$2 \cdot 2^2$

$3 \cdot 2^1$

$4 \cdot 2^0$

5×1

$$2^{n-1} \cdot 1 + 2^{n-2} \cdot 2 + \cdots + 2^{n-n} \cdot n + n+1$$

average...

$$\mathcal{I}_{f,n} = \{i \mid i \text{ is an input to } f \wedge |i| = n\}$$

```
def has_even(number_list):
    for number in number_list:
        if number % 2 == 0:
            return True
    return False
```

$$\sum_{i=1}^{n} i 2^{n-i} = 2^n \sum_{i=1}^{n} i 2^{-i} = 2^n \sum_{i=1}^{n} i \left(\frac{1}{2}\right)^i$$

summation...

$$\sum_{i=0}^{n-1} i r^i = \sum_{i=1}^{n} (i-1) r^{i-1} = \sum_{1}^{n} i r^{i-1} - \sum_{1}^{n} r^{i-1}$$

$$= \frac{1}{r}\sum_{1}^{n} i r^i - \frac{1}{r}\sum_{1}^{n} r^i$$

$$\frac{1}{r}\sum_{1}^{n} r^i = \frac{1}{r}\sum_{1}^{n} i r^i - \sum_{0}^{n-1} i r^i$$

$$\sum_{i=0}^{i=n-1} i r^i = \frac{n r^n}{r \quad 1} + \frac{r \quad r^{n+1}}{(r \quad 1)^2}$$

$$1 \cdot \sum_{i=1}^{n} r^i = r + r^2 + \dots + r^n$$

$$r \sum_{i=1}^{n} r^i = r^2 + \dots + r^n + r^{n+1}$$

$$(r-1) \sum_{i=1}^{n} r^i = r^{n+1} - r \Rightarrow \sum_{i=1}^{n} r^i = \frac{r^{n+1} - r}{r-1} \quad //$$

summation...

$$\sum_{i=1}^{n} i\left(\frac{1}{2}\right)^{i}$$

$$\sum_{i=0}^{i=n+1} ir^{i} = \frac{(n+1)r^{n+1}}{r \quad 1} + \frac{r \qquad r^{n+2}}{(r \quad 1)^{2}}$$

$$-\frac{(n+1)\left(\frac{1}{2}\right)^{n+1}}{} + \frac{\frac{1}{2} - \left(\frac{1}{2}\right)^{n+2}}{\left(\frac{1}{2}\right)^{2}}$$

$$= 2 - (n+1)\left(\frac{1}{2}\right)^{n} - \left(\frac{1}{2}\right)^{n} = 2 - \frac{n+2}{2^{n}} + \frac{n+1}{2^{n}}$$

$$= 2 - \frac{1}{2^{n}}$$

finding a needle...

...when you know it's in the haystack

```
# num_list is a list of numbers,
# a permutation of {1, 2, 3, ..., n}
def find_one(num_list):
    for i in range(len(num_list)):
        if num_list[i] == 1:
            return i
```

(handwritten: "natural" inserted before "numbers"; $n!$ and "permutation" pointing at the list)

$$\Sigma = 1 + 2 + \cdots + n$$
$$\Sigma = n + n-1 + \cdots + 1$$
$$2\Sigma = n(n+1) \rightarrow \Sigma = \frac{n(n+1)}{2}$$

$(n-1)! \cdot 1$

$(n-1)! \cdot 2$

$\vdots$

$(n-1)! \cdot n$

$$\frac{\sum_{i=1}^{n} i(n-1)!}{n!} = \frac{(n-1)!}{n!}\sum_{i=1}^{n} i = \frac{1}{n}\cdot\frac{n(n+1)}{2} = \frac{n+1}{2}$$

graphs (discrete ones)...

what can you do with them?

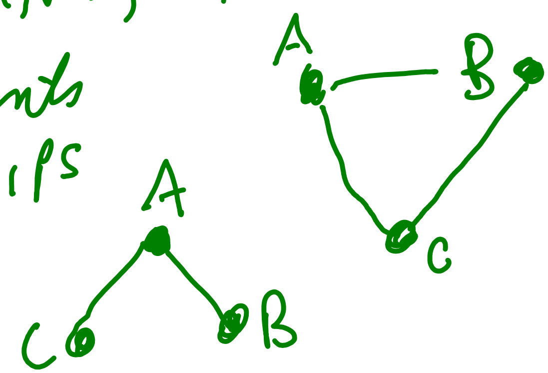
V – some finite set

E – some pairs from i.e. (u,v), $u,v \in V$

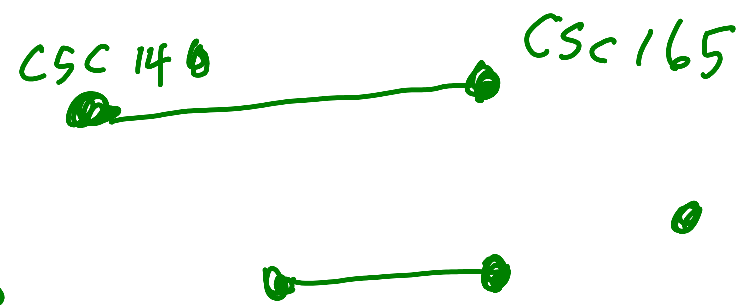
$|V| \sim 2B$

V = facebook accounts

E – friendships



- represent friendships



- represent lecture sections

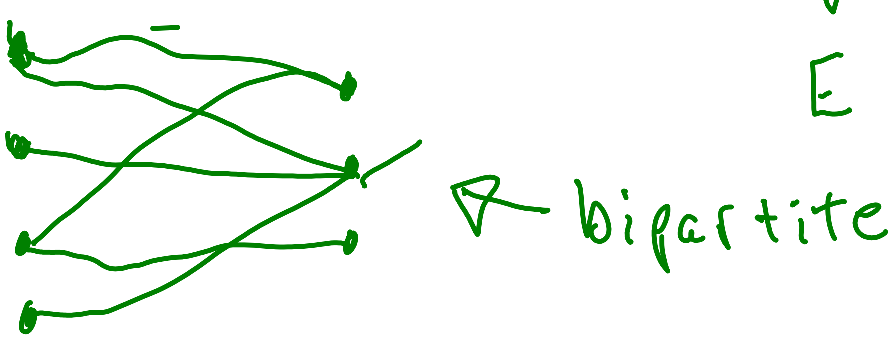
V = {course lecture sections}

E = {(c_1, c_2) | c_1, c_2 at same time}

- represent tasks↔person

V = {people ∨ jobs}

E = {(p, j) | p qualified to do j}



definitions...

$G = (V, E) \in \mathcal{G}$ → simple finite graphs

$|V| \in \mathbb{N}$

$|E| \in \mathbb{N}$

$(u,v) = (v,u)$

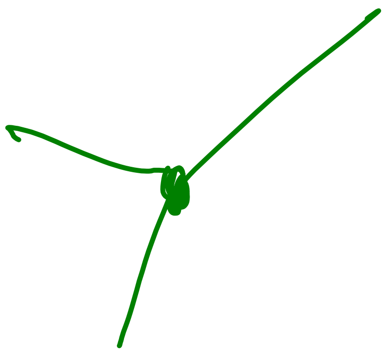
- no "loops", i.e. $u \in V$, $(u,u) \notin E$
- at most 1 edge per pair

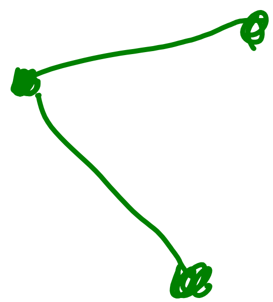
for $|V| = n$, min # of edges?

$$\frac{n(n-1)}{2} \geq |E|$$

degree, degree-sum, max number of edges?

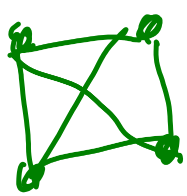
$d(v)$: $|\{(v,u) \mid (v,u) \in E\}|$, for $v \in G = (V, E)$

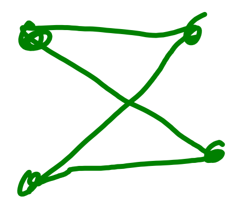




degrees: affect cycles.

degree-sum $G=(V,E) = \sum_{v \in V} d(v)$

$G =$ 

$G' =$ 

degree-sum$(G) = 12$

degree-sum$(G') = 8$

paths, connectedness... in $G = (V, E)$

A path from u to v: Distinct vertices $v_0, \ldots, v_k$ in V where:

- $u = v_0$, $v = v_k$
- if $0 \le i \le k$ 1, then $(v_i, v_{i+1}) \in E$

We allow $k = 0$ there is a path from v to itself

path length from u to v: number of edges in path from u to v

u, v are connected: There is a path from u to v.

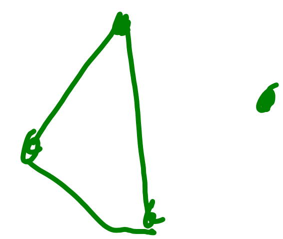
graph G is connected: every pair $u, v \in G$ is connected

$\forall n \in \mathbb{N}, \exists M \in \mathbb{N}, \forall G = (V, E), (V = n \wedge E \geq M) \Rightarrow G$ is connected?

$G = (V, E) \wedge |V| = n$ $\quad \frac{n(n-1)}{2}$ guarantees

could have as few a $n-1$ edges + have connected, e.g. linked lists trees.

would $2n$ edges be enough?



n^2 edges $\quad n^2 > \frac{n^2 - n}{2}$

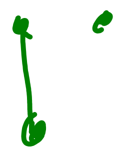
Any graph with n^2 edges, $|V| = n$ is connected — vacuous truth

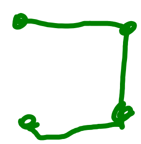
Notes

How about $\frac{(n-1)(n-2)}{2}$ edges

$n = 1$? → not defined

$n = 2$? → not connected

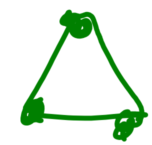
$n = 3$? →  not connected

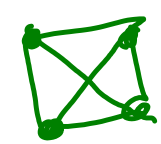
$n = 4$?  maybe if $n \geq 4$???

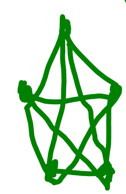
Claim $\forall n \in \mathbb{N}, \exists G = (V, E), |V| = n \geq 4 \wedge |E| = \frac{(n-1)(n-2)}{2}$

$\wedge$ G is not connected

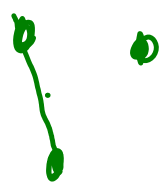
C_k : complete graph on k vertices

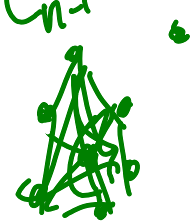
e.g. C_3 : 

C_4 : 

C_5 : 

C_1 

C_2 

C_{n-1} 

Notes

$$\forall n \in \mathbb{N},\ n>1,\ \forall G=(V,E),\ |V|=n \wedge |E| = \frac{(n-1)(n-2)}{2}+1$$

$$\Rightarrow G \text{ is connected.}$$