average...

$$Ave(RT(n)) = \frac{\sum_{i \in \mathcal{I}_{f,n}} RT_{f(i)}}{|\mathcal{I}_{f,n}|}$$

$$\mathcal{I}_{f,n} = \{i \mid i \text{ is an input to } f \wedge |i| = n\}$$

Lists of 0s + 1s.

```
def has_even(number_list):
    for number in number_list:
        if number % 2 == 0:
            return True
    return False
```

- calculated.
- # steps based on list length n
- 2^n lists of length n

$$2^{n-1} \cdot 1 + 2^{n-2} \cdot 2 + \cdots + 2^{n-n} \cdot n = \sum_{i=1}^{n} i\,2^{n-i}$$

+ (n+1)]

summation...

$$\sum_{i=1}^{n} i\,2^{n-i} = 2^n \sum_{i=1}^{n} i\,2^{-i} = 2^n \sum_{i=1}^{n} i\left(\frac{1}{2}\right)^i$$

$2^{-i} = \left(\frac{1}{2}\right)^i$

$$\sum_{i=0}^{n-1} i r^i = \sum_{i=1}^{n} (i-1) r^{i-1} = \sum_{i=1}^{n} i r^{i-1} - \sum_{i=1}^{n} r^{i-1} = \frac{1}{r}\left(\sum_{i=1}^{n} i r^i - \sum_{i=1}^{n} r^i\right)$$

exercise

$$\sum_{i=0}^{i=n-1} i r^i = \frac{n r^n}{r \quad 1} + \frac{r \quad r^{n+1}}{(r \quad 1)^2}$$

formula: last thing Ch 5.

finding a needle...

...when you know it's in the haystack

```
# num_list is a list of  numbers,
# a permutation of {1, 2, 3, ..., n}
def find_one(num_list):
    for i in range(len(num_list)):
        if num_list[i] == 1:
            return i
```

by the way $0! = 1$

$n!$

$(n-1)! \cdot 1 \quad (0+1)$

$(n-1)! \cdot 2 \quad (0+2)$

$\vdots$

$(n-1)! \cdot n$

$$\frac{\sum_{i=0}^{n-1} (i+1) \cdot (n-1)!}{n!} = \frac{(n-1)!}{n!} \sum_{i=1}^{n} i = \frac{1}{n} \frac{n(n+1)}{2} = \frac{n+1}{2} \in \theta(n)$$

graphs (discrete ones)...

what can you do with them?

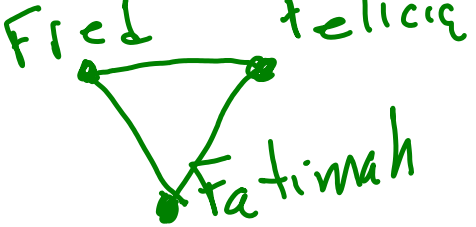
= finite sets

= { users }

V = { facebook friendships }

E = { facebook

$|V| \approx 2B$

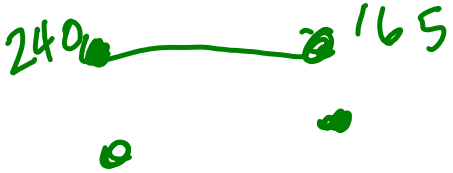


- represent friendships

- represent lecture sections

V = { course lecture sections }

E = { $(c_1, c_2) \mid c_1, c_2 \in V \wedge$ c_1 c_2 at same time }



- represent tasks↔person

$V = P \cup T$, P = { people } T = { tasks }

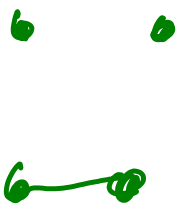
E = { $(p,t) \mid p \in P \wedge t \in T \wedge p$ qualified to t }

definitions...

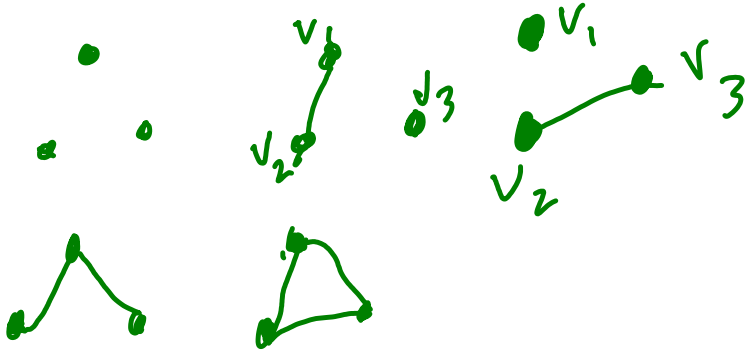
$G = (V, E) \in \mathcal{G}$

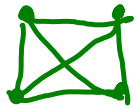
↑ vertices edges

$|V| = 1$ $|V| = 2$

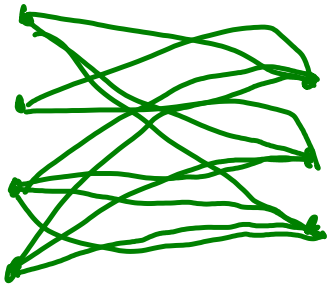


$\mathcal{G}$ includes only finite graphs with no "loops" (no edge (v, v)). at most 1 edge from $v \leftrightarrow u$ (simple graphs)





C_4



$K_{4,3}$

(complete bipartite graph 4,3)