big-Oh hierarchy

$a, b \in \mathbb{R}^+$

$g \in O(f) = \{g : g : \mathbb{N} \to \mathbb{R}^+, \exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \leq c f(n)\}$

$\log_a n$ versus $\log_b n$ (logarithmic) $\to a, b > 1$

n^a versus n^b (polynomial)

a^n versus b^n (exponential)

$\log_a n$ versus n^a

n^a versus b^n

explore!

$a^{\log_a n} = n = b^{\log_b n}$

$\log_a n = \log_a b \cdot \log_b n$

$f(n) = \log_b n$, $g(n) = \log_a n$

then choose $c = \log_a b$, $n_0 = 1$

special case

$n^a \in O(\log_a n)$

$\notin \Omega(\log_a n)$.

assume $b > a > 0$

$n^a \in O(n^b)$ but $n^a \notin \Omega(n^b)$

$1 < a < b$

$a^n \in O(b^n)$ but $a^n \notin \Omega(b^n)$ eg $b = 1.\varepsilon \times a$

then $b^n = a^n \cdot \underbrace{\quad}_{n}$ $\uparrow$

$a > 1$ $\log_a n \in O(n^a)$ but $n^a \notin O(\log_a n)$

$n^a \in O(b^n)$ but $b^n \notin O(n^a)$

$a, b > 1$

negation of big-Oh

$\forall c, n_0 \in \mathbb{R}^+, \exists n \in \mathbb{N}, n \geq n_0 \wedge g(n) > c f(n)$

properties

- reflexivity $f \in O(f)$ .. $c, n_0 = 1, 1$

- transitivity of big-Oh $g \in O(f) \wedge f \in O(h) \Rightarrow g \in O(h)$

<u>discuss</u> $\exists n_1, c_1 \in \mathbb{R}^+ \ldots n \geq n_1,\ g(n) \leq c_1 f(n)$ | <u>want</u> $c_3 = c_1 \cdot c_2$, $n_3 = n_1 + n_2$

$\exists n_2, c_2 \in \mathbb{R}^+ \ldots n \geq n_2,\ f(n) \leq c_2 h(n)$ | $g(n) \leq c_3 h(n)$

- not symmetry (anti-symmetry...)

~~$g(n) \in \Theta$~~ ($g \in O(f)$ does NOT imply $f \in O(g)$

it does imply $f \in \Omega(g)$

products and sums

$a \in \mathbb{R}^+ \quad af(n) = a \times f(n)$

- af $\quad f \in O(g) \Rightarrow af \in O(g)$

<u>discuss</u> $n_0, c \in \mathbb{R}^+ \ldots \quad n \geq n_0 \Rightarrow f(n) \leq c\,g(n)$

let $c_1 = ac \quad \ldots\ldots \quad af(n) \leq c_1 g(n)$

- $f \cdot g$ $\quad f \in O(h_1),\ g \in O(h_2)$

$f \cdot g \in O(h_1 h_2)$

- $f + g$ $\quad (f+g)(n)$ means $f(n) + g(n)$

$g \in O(f) \wedge f \in O(h) \Rightarrow f+g \in O(h) \quad f+g \in \Omega(f)$

<u>Notice</u> same assumptions so n_0, c_0, n_1, c_1 exist

<u>discussion</u> by <u>trans</u> $g \in O(h)$

$\forall n \geq n_0 \quad g(n) \leq c_1 h(n)$

$f(n) \leq c_2 h(n)$