reading: just browse number rep: understand main theorem

CSC165 fall 2017

begin algorithm analysis

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Using Course notes: more Induction

Outline

notes

time resource — really care

measure versus "wall clock"

available

How much time does this take?

```
def f(list_):
    for i in list_:
        print(i)
```

depends (linearly) on $\boxed{len(list\_) = n}$

depends on print function

independent of n

depend on processor

depend on language

Ram

size of each i

assumptions, assumptions...

- "steps" — independent of input size
  $x > y$ $x * y$ len(s)
  $a = 15$ constant — often 1
- ignore constant factors
  Comparison up to constant factor
- ignore "noise" for small input → near size 0

We care about growth rate of time consumption

formalizing assumptions

express run time as function

$f : \mathbb{N} \to \mathbb{R}^+$

- f absolutely dominates g

Let $f, g : \mathbb{N} \to \mathbb{R}^+, \forall n \in \mathbb{N}, f(n) \geq g(n)$

- f dominates g up to a constant factor — $\left.\begin{array}{l} f(n) = 10n \\ g(n) = 157n \end{array}\right] c = 15.7$

Let $f, g : \mathbb{N} \to \mathbb{R}^+, \exists c \in \mathbb{R}^+, \forall n \in \mathbb{N}, \; cf(n) \geq g(n)$

- f eventually dominates g up to a constant factor

Let $f, g : \mathbb{N} \to \mathbb{R}^+, \exists c \in \mathbb{R}^+, \exists n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow cf(n) \geq g(n)$

What should domain and range of f, g be?

$f(n) = 10n \; \cancel{+ 13{,}000{,}000}$

$g(n) = 157n + 13000000$

big-Oh, big-Omega, big-Theta

... and you're started on the Greek alphabet... σ θ Θ · O — not Θ !!!

"g is big-Oh", $g \in \cancel{\Theta}O(f)$: $g \in \{h : \mathbb{N} \to \mathbb{R}^+ \mid \exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow h(n) \leq cf(n)\}$

"g is big Omega of f"

$g \in \Omega(f)$: $g \in \{h : \mathbb{N} \to \mathbb{R}^+ \mid \exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow h(n) \geq cf(n)\}$

"g is big-Theta of f"

$g \in \Theta(f)$: $g \in \{h : \mathbb{N} \to \mathbb{R}^+ \mid \exists c_1, c_2, n_0 \in \mathbb{R}, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow h(n) \geq c_1 f(n) \wedge h(n) \leq c_2 f(n)\}$

big-Oh hierarchy

$\log_a n$ versus $\log_b n$ (logarithmic)

n^a versus n^b (polynomial)

a^n versus b^n (exponential)

$\log_a n$ versus n^a

n^a versus b^n

explore!

$$10^{\log x} = x = 2^{\lg_2 x}$$

$$\Rightarrow \log_{10} x = \log_{10} 2 \cdot \lg_2 x$$

$$\frac{\log_{10} x}{\log_{10} 2} = \lg_2 x$$