test solutions: up tomorrow...

CSC165 fall 2017

Mathematical expression:
induction: different starting points, etc.

Danny Heap
csc16517f@cs.toronto.edu
BA4270 (behind elevators)
Web page:
http://www.teach.cs.toronto.edu/~heap/165/F17/
416-978-5899

Using Course notes: Induction

Outline

notes

review induction parts

- claim — predicate of natural numbers named (possibly), e.g. $P(n)$
  want to prove: $\forall n \in \mathbb{N}, P(n)$
  - base case — point where claim begins to be true, so far 0
  - inductive step $\forall n \in \mathbb{N}, P(n) \Rightarrow P(n+1)$ (entire implication)

more domino logistics....

$7^0 \equiv 1 \pmod 6$ $\quad 7^1 \equiv 1 \pmod 6$ $\quad 7^2 \equiv 1 \pmod 6$ $\quad 7^3 \equiv 1 \pmod 6$ $\quad 7^4 \equiv 1 \pmod 6$ $\quad 7^5 \equiv 1 \pmod 6$ $\quad \cdots$ "etc."

don't fall

$7^0 \equiv 1 \pmod 6$ $\quad 7^1 \equiv 1 \pmod 6$

but, starts to fall here...

$7^2 \equiv 1 \pmod 6$ $\quad 7^3 \equiv 1 \pmod 6$ $\quad 7^4 \equiv 1 \pmod 6$ $\quad 7^5 \equiv 1 \pmod 6$ $\quad \cdots$ "etc."

$n>2 \Rightarrow$ $2n+1 < 2^n$: $P(n)$ — for $n \in \mathbb{N}$

Claim: ~~$\forall n \in \mathbb{N}, n > 2 \Rightarrow 2n+1 < 2^n$~~ $\forall n \in \mathbb{N}, P(n)$

Proof (by induction)

base case: $2(3)+1 = 7 < 8 = 2^3$. So $P(3)$ is true.

Inductive step Let $k \in \mathbb{N}$. Assume $P(k)$: $k > 2$ $\Rightarrow 2k+1 < 2^k$ (IH). Want to show

$P(k+1)$: if $k+1 > 2$ then $2(k+1)+1 < 2^{k+1}$.

Assume $k \geq 3$ (otherwise claim trivially true)

$2n + 1 < 2^n$

$$
\begin{aligned}
2(k+1)+1 &= 2k+2+1 \\
&< 2^k + 2^k \quad (\because 2^k > 2k+1 \text{ by IH and } 2^k \geq 8 > 2 \text{ for } k \geq 3) \\
&= 2^{k+1}
\end{aligned}
$$

$\blacksquare$

$\forall n \in \mathbb{N}, 3^n \geq n^3$

Proof (induction)

base cases $3^0 = 1 \geq 0 = 0^3$, $3^1 = 3 \geq 1 = 1^3$, $3^2 = 9 \geq 8 = 2^3$, $3^3 = 27 \geq 27 = 3^3$. So $P(0)$, $P(1)$ and $P(2)$ and $P(3)$ are true.

Inductive step Let $k \in \mathbb{N}$. assume $k \geq$ assume $P(k)$, $3^k \geq k^3$. Must show that $P(k+1)$ follows: $3^{k+1} \geq (k+1)^3$.

So 3^{k+1} $= 3 \times 3^k = 3^k + 3^k + 3^k$

$\geq k^3 + k^3 + k^3$ (IH)

$\forall n \in \mathbb{N}, 3^n \geq n^3$

$k \geq 3$

$\geq k^3 + 3k^2 + 9k$ (since $k \geq 3$.) $\quad k^2 \geq 3k$

$= k^3 + 3k^2 + 3k + 6k \quad k^3 \geq 3k^2 \geq 9k$

$\geq k^3 + 3k^2 + 3k + 1$ (since $k \geq 3 > 1/6$)

$= (k+1)^3$

$3k^3 \geq (k+1)^3$

$\forall x, y \in \mathbb{N}, \forall n \in \mathbb{N}, x - y \quad x^n - y^n$

order of introductions...

Notes

Want $2(n+1)+1 < 2^{n+1} = 2 \cdot 2^n$

assume $n \geq 3$. and $2n+1 < 2^n$

$2n + 2 + 1 \quad \ldots \quad 2^n + 2^n$

Notes

Assume $n \geq 3$ and $3^n \geq n^3$

$n^2 \cdot n \geq 3n^2$

$3^{n+1} = 3 \times 3^n = 3^n + 3^n + 3^n$

$\geq n^3 + n^3 + n^3$

$\vdots$

$(n+1)^3 = n^3 + 3n^2 + 3n + 1$

$n \geq 3$

$3n \geq 9$