

generalize  $n \mid n+3 \Rightarrow n \mid d$  for  $n \in \mathbb{N}$

$\forall n, d \in \mathbb{Z}, n \mid n+d \Rightarrow n \mid d$

exercise revisit last Thurs proof

$\mathbb{N} \rightarrow \mathbb{Z}$

$3 \rightarrow d$

$\forall n \rightarrow \forall n, d$

Now suppose you've done  $\uparrow$ , and  
choose  $d$  such that  $\text{prime}(d)$ .

$\forall n, d \in \mathbb{N}, \text{prime}(d) \wedge n \mid n+d \Rightarrow n \mid d$



prove  $m, n \equiv 1 \pmod{3} \Rightarrow mn \equiv 1 \pmod{3}$

$a \equiv b \pmod{k}$ : " $k \mid a-b$ ", for  $a, b, k \in \mathbb{Z}, k \neq 0$ .

Suppose  $\begin{array}{rcl} a \equiv 1 & \dots & 2 \\ a \equiv 1 & \pmod{3} \end{array}$

translation:  $\forall m, n \in \mathbb{Z}, [\exists i (m-i) \wedge (3 \mid n-i)] \Rightarrow 3 \mid (mn-1)$

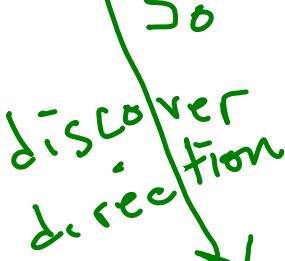
discussion  $\exists k_1 \in \mathbb{Z}, \underline{m-1=3k_1}, m=3k_1+1$

$\exists k_2 \in \mathbb{Z}, n-1=3k_2, n=3k_2+1$

So  $mn-1 = \underline{9k_1k_2+3k_1+3k_2}$

$= 3(\underline{3k_1k_2+k_1+k_2})$

Let  $k_3 \parallel$

discover direction 

proof direction 



prove  $m, n \equiv 1 \pmod{3} \Rightarrow mn \equiv 1 \pmod{3}$

Proof Let  $m, n \in \mathbb{Z}$ . Assume  $3 \mid m-1$ , that is  $\exists k_1 \in \mathbb{Z}$  such that  $3k_1 = m-1$ . Let  $k_1$  be such a value. Also assume  $3 \mid (n-1)$ , that is  $\exists k_2 \in \mathbb{Z}$ ,  $3k_2 = n-1$ . Let  $k_2$  be such a value. Let  $k_3 = 3k_1k_2 + k_1 + k_2$ .

$$\begin{aligned} \text{Then } 3k_3 &= 9k_1k_2 + 3k_1 + 3k_2 \\ &= (3k_1 + 1)(3k_2 + 1) - 1 \\ &= m \cdot n - 1 \quad \blacksquare \end{aligned}$$

converse of previous example?

$$\forall m, n \in \mathbb{Z}, (3|m_n - 1) \Rightarrow 3|m - 1 \wedge 3|n - 1$$

True requires argument using generic  
integers  $m, n$  7 count

False requires Counter. counter example

Let  $m, n = 2$  and  $3|4 - 1$

contrapositive  $P \Rightarrow Q$  c.p.  $\neg Q \Rightarrow \neg P$   
 $\Rightarrow$  equivalent  $\Rightarrow$

$$\forall m, n \in \mathbb{Z}, 3 \nmid m - 1 \Rightarrow 3 \nmid m - 1 \vee 3 \nmid n - 1$$