generalize $n \quad n + \underset{d}{\cancel{3}} \Rightarrow n \quad \underset{d}{\cancel{3}}$ for $n \in \mathbb{N}$

$\forall n, d \in \mathbb{Z},\ n \mid n+d \Rightarrow n \mid d$

<u>exercise</u> revisit last Thurs proof

$\mathbb{N} \to \mathbb{Z}$

$3 \to d$

$\forall n \to \forall n, d$

Now suppose you've done ↑, and choose ~~3~~ d such that Prime(~~3~~ d).

$\forall n, d \in \mathbb{N},\ \text{Prime}(d) \wedge n \mid n+d \Rightarrow n \mid d$

prove $m, n \equiv 1 \bmod 3 \Rightarrow mn \equiv 1 \bmod 3$

$a \equiv b \bmod k$: "$k \mid a-b$", for $a, b, k \in \mathbb{Z}$, $k \neq 0$.

suppose $a \equiv 1 \bmod 2$

$a \equiv 1 \bmod 3$

translation: $\forall m, n \in \mathbb{Z}, [3 \mid (m-1) \wedge (3 \mid n-1)] \Rightarrow 3 \mid (mn-1)$

discussion $\exists k_1 \in \mathbb{Z}$, $\underline{m-1 = 3k_1}$, $m = 3k_1 + 1$

$\exists k_2 \in \mathbb{Z}$, $n-1 = 3k_2$, $n = 3k_2 + 1$

So $mn - 1 = 9k_1k_2 + 3k_1 + 3k_2$

$= 3(\underbrace{3k_1k_2 + k_1 + k_2)}_{\text{let } k_3}$ //

discover direction

proof direction

prove $m, n \equiv 1 \bmod 3 \Rightarrow mn \equiv 1 \bmod 3$

Proof Let $m, n \in \mathbb{Z}$. Assume $3 \mid m-1$, that $\exists k_1 \in \mathbb{Z}$ $3k_1 = m-1$. Let $k_1$ be such a value. Also assume $3 \mid (n-1)$, that is $\exists k_2 \in \mathbb{Z}$, $3k_2 = n-1$. Let $k_2$ be such a value. Let $k_3 = 3k_1k_2 + k_1 + k_2$

Then
$$
\begin{aligned}
3k_3 &= 9k_1k_2 + 3k_1 + 3k_2 \\
&= (3k_1+1)(3k_2+1) - 1 \\
&= m \cdot n - 1 \quad \blacksquare
\end{aligned}
$$

# converse of previous example?

$\forall m, n \in \mathbb{Z}, (3 \mid mn-1) \Rightarrow 3 \mid m-1 \wedge 3 \mid n-1$

True requires argument using generic integers $m, n$

False requires counter- ] counter example

let $m, n = 2$ $mn = 4$ and $3 \mid 4-1$

contrapositive $P \Rightarrow Q$ c.p. $\neg Q \Rightarrow \neg P$

equivalent

$\forall m, n \in \mathbb{Z}, 3 \nmid mn-1 \Rightarrow 3 \nmid m-1 \vee 3 \nmid n-1$