PS1 - posted tonight ... 6 office hours + piazza ...

CSC165 fall 2017

Mathematical expression: proofs

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Using Course notes: Proof

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Outline

defining, unpacking

existential

universal

notes

$a \quad b$ and $Prime(p)$

statements packed and unpacked

"a divides b": "$\exists k \in \mathbb{Z},\ b = ka$, write $a|b$", where $a, b \in \mathbb{Z}$

$$\forall n \in \mathbb{N},\ n|5 \Rightarrow n|10.$$

$$\forall n \in \mathbb{N},\ (\exists k_1 \in \mathbb{Z},\ 5 = k_1 n) \Rightarrow (\exists k_2 \in \mathbb{Z},\ 10 = k_2 n)$$

$Prime(p)$: "$\underbrace{p > 1 \wedge \forall n \in \mathbb{N},\ n|p \Rightarrow (n = 1 \vee n = p)}$"

"$p > 1 \wedge \forall n \in \mathbb{N},\ (\exists k \in \mathbb{Z},\ p = kn) \Rightarrow (n = 1 \vee n = p)$

prove 3 18 — $\exists k \in \mathbb{Z},\ 3k = 18$ — translation

discussion: why?

Proof

Let $k = 6$. # header

Then $\underline{3 \cdot k =} 3 \cdot 6 = 18$ $\blacksquare$ # body

There is a real solution to $x^2 + 2x + 3 = 11$

$\exists x \in \mathbb{R}, x^2 + 2x + 3 = 11$ # translation

Proof

Let $x = -4$. #header

Then $x^2 + 2x + 3 = 16 - 8 + 3$
$= 11$ $\blacksquare$

prove $n^2 + 2n + 5 > 4$ if $n \in \mathbb{N}$

$\forall n \in \mathbb{N}, n^2 + 2n + 5 > 4$ # trans.

Proof

Let $n \in \mathbb{N}$ (Let n be a fixed, arbitrary element of $\mathbb{N}$) #head.

Then $5 > 4$

$n^2 + 2n + 5 \geq 5 > 4$ # $n, n^2 \geq 0$, $n \in \mathbb{N}$

logic - from known → deductions

prove $n^2 - 5n > 7$ for most $n \in \mathbb{N}$

$\forall n \in \mathbb{N}, n > 6 \Rightarrow n^2 - 5n > 7$ # trans

Proof

try $n = 115$

Let $n \in \mathbb{N}$. Assume $n \geq 7$. ← same as $n > 6$ for integers...

$n^2 - 5n \geq 7^2 - 5 \cdot 7$ # $n \geq 7$

$n(n-5)$ $= 14 > 7$

Use calculus + check derivative!

– increasing function when $n > \frac{5}{2}$

prove $n^2 - 5n > 7$ for most $n \in \mathbb{N}$

another approach ...

$n \geq 7$

$n - 5 \geq 7 - 5 = 2$

$n(n-5) = n^2 - 5 \geq 2n$ *

also

$n \geq 7$ **

$2n \geq 14$

$n^2 - 5 \geq 2n \geq 14 > 7$

combine * and **

prove $n \quad n+3 \Rightarrow n \quad 3$ for $n \in \mathbb{N}$

$$\forall n \in \mathbb{N}, \left(\exists k_1 \in \mathbb{Z}, (n+3) = k_1 n\right) \Rightarrow \left(\exists k_2 \in \mathbb{Z}, 3 = k_2 n\right)$$

Let n be an arbitrary $\in \mathbb{N}$. Assume $\exists k_1 \in \mathbb{Z};\ n+3 = k_1 n$. Let $k_2 = k_1 - 1$

how did we think of this k_2?

Then
$$\begin{aligned} n+3 &= k_1 \cdot n \\ &= (k_2+1)n \\ &= k_2 n + n \end{aligned}$$

So $3 = k_2 n$ $\square$

prove $n \quad n + 3 \Rightarrow n \quad 3$ for $n \in \mathbb{N}$

want $k_2 n = 3$

discovery direction

$k_2 n + 3 = 3 + n = k_1 n$

$k_2 n = k_1 n - n$
$= (k_1 - 1) n$

let $k_2 = k_1 - 1$

logic direction

(use this in proof)