CSC165 fall 2017

graph connectivity, trees

Danny Heap
csc16517f@cs.toronto.edu
BA4270 (behind elevators)
Web page:
http://www.teach.cs.toronto.edu/~heap/165/F17/
416-978-5899

Using Course notes: average analysis; graphs

Outline

notes

must be connected

$\forall n \in \mathbb{N}, \exists M \in \mathbb{N}, \forall G = (V, E), (|V| = n \wedge |E| \geq M) \Rightarrow G$ is connected?

maybe be connected

$\forall n \in \mathbb{N}, \exists G = (V, E), |V| = n \wedge |E| = n - 1 \wedge G$ is connected

must be disconnected

$\forall n \in \mathbb{N}, \forall G = (V, E), (|V| = n \wedge |E| \leq n - 2) \Rightarrow G$ is not connected

steps:

- natural to reason by removing an edge from a connected graph with $n - 1$ edges...
- first need some results about which components of connected graphs have redundant edges (cycles)...
- then need some results about connected graphs without cycles (trees)...
- then reason about reducing an arbitrary connected graph to a tree...

whew!

cycle

consecutively adjacent vertices $v_0, \ldots, v_k \in V \wedge k \geq 3$,

all distinct except $v_0 = v_k$

$\forall G = (V, E), \forall e \in E, G$ connected $\Rightarrow (e$ in a cycle of $G \Leftrightarrow G - e$ connected $)$

tree: connected, acyclic graph

removing any edge from a tree disconnects it

$\forall G = (V, E), G \text{ is a tree } \Rightarrow |E| = |V| - 1$

but first...

$\forall G = (V, E), (G \text{ is a tree } \wedge |V| \geq 2) \Rightarrow (\exists v \in V, d(v) = 1)$

main result...

$\forall n \in \mathbb{N}^+, \forall G = (V, E), (G \text{ is a tree } \wedge |V| = n) \Rightarrow |E| = |V| - 1$

big picture...

Notes