CSC165 fall 2017

Mathematical expression: predicate logic — reading

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Using Course notes: Mathematical Expression: predicate logic

Outline

bi-implication

predicates

quantifiers

multiple quantifiers

mixed quantifiers

negation

number theory intro

notes

annotated slides

compare and contrast...

"If it rains, then I will wear sneakers." $r \Rightarrow s$

"If and only if it rains, then I will wear sneakers." $r \Leftrightarrow s$

$r \wedge s \Rightarrow T \Leftrightarrow T$

$r \wedge \neg s \Rightarrow F \Leftrightarrow F$

$\neg r \wedge s \Rightarrow T \Leftrightarrow F$

$\neg r \wedge \neg s \Rightarrow T \Leftrightarrow T$

what's a predicate?

boolean function $\rightarrow \{T, F\}$

$n > 7.2$

x is tall

domain

$n = 5$ — F

$n = 11$ — T

$n =$ "fred" → WTF ???

$n > 7.2 : \mathbb{R} \rightarrow \{T, F\}$

Danny is tall → F

$x = KK$ → T

$x = 7$ → ???

x is tall : Animal → $\{T, F\}$

predicate definitions — naming, use symbols

IsTall: "x is tall", where x is an animal

$G(n)$: "$n > 7.2$", where $n \in \mathbb{N}$

quantifiers $\forall$ and $\exists$

$n > 7.2$ depends on n

$\forall n \in \mathbb{N}, n > 7.2$

means $0 > 7.2 \wedge 1 > 7.2 \wedge 2 > 7.2 \wedge 3 > 7.2 \wedge \cdots$

False because counterexample

0 is not > 7.2 makes

entire $\wedge \cdots \wedge$ False.

$\exists n \in \mathbb{N}, n > 7.2$

means $0 > 7.2 \vee 1 > 7.2 \vee 2 > 7.2 \vee \cdots$

True - just need one example,

e.g. $8 > 7.2$

translate quantified predicates

$\forall n \in \mathbb{N}, n > 7.2$

For every natural number n, n is greater than 7.2

Every natural number is greater than 7.2.

$\exists n \in \mathbb{N}, n > 7.2$

There exist some natural number n, where (such that) n is greater than 7.2

There is a natural number greater than 7.2

quantified binary predicates

$x + y = 17$: $\mathbb{Z} \times \mathbb{Z} \to \{T, F\}$

$\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x + y = 17$ ↘ equivalent

$\forall x, y \in \mathbb{Z}, x + y = 17$

False, eg $x = 3, y = 4$, $x + y \neq 17$

Because both quants. are $\forall$ (same)

means same as $\forall y, x \in \mathbb{Z}, x + y = 17$

$\forall y, \in \mathbb{Z} \forall x \in \mathbb{Z}, x + y = 17$

$\exists x, y \in \mathbb{Z}, x + y = 17$

True ex $x = 16, y = 1$

Same as $\exists y \in \mathbb{Z}, \exists x \in \mathbb{Z}, x + y = 17$

multiple quantifier examples

done

order matters!

$x + y = 17$

$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x + y = 17$

True read left to right: fix arbitrary $x \in \mathbb{Z}$, then choose/pick $y = 17 - x$, then $x + y = x + (17 - x) = 17$

$\exists y \in \mathbb{Z}, \forall x \in \mathbb{Z}, x + y = 17$

False y is chosen before x, this claims there is some y with

$0 + y = 17$

$1 + y = 17$

$-1 + y = 17$