CSC165 fall 2017

Mathematical expression

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Using Course notes: Prologue, Mathematical Expression

Outline

Introduction

sets

functions

sums and products

propositional logic

notes

annotated slides

what's CSC165?

a course about expression (communication):

- with and through programs — express solutions to problems
- with developers — documentation
- knowing what **you** mean — explain code over the phone!
- understanding what **others** mean
  - manuals!
  - specifications
- analyzing arguments, programs
  - correct
  - efficient

CS needs math:

- graphics — geometry, linear algebra
- verification — logic
- cryptography — number theory — RSA public key
- artificial intelligence — probability
- complexity — calculus
- numerical analysis — calculus
- networking stats
- databases set theory

⋮

doing well in CSC165

csc165 gateway for our Post

Doing well has two aspects: one being recognized as doing well by being awarded credit (grades), another being able to retain concepts and tools for use later on. Here's how to do both:

- Read the course web page, and emails, regularly. Understand the course information sheet.
- Spend enough time. We assume an average of 8 hours/week 4 in lecture/problem sessions, 4 reviewing preparing assignments
  2+2
- Ask questions. Make your own annotations.

balance

- computers are precise   in identical environments they execute identical instructions identically
- humans are as precise as necessary, and different human audiences require different levels of precision
- The *really* difficult job is finding the right level of precision. Too much precision introduces unbearable tedium; too little introduces unfathomable ambiguity.
- Proofs are primarily works of literature: they communicate with humans, and the best proofs have suspense, pathos, humour and surprise. As a side-effect, proofs present a convincing argument for some fact.

building sets...in math

English prose

The set of students in FG103 with letter "q" as second letter of their surname — $\{\}$, $\emptyset$

list elements

$S = \{1, 3, 9, 7\}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

$\mathbb{Z}^+, \mathbb{N}^+ = \{1, 2, 3, 4, \ldots\}$

set comprehension

$\{x \mid x \in \mathbb{N} \text{ and } x > 3\}$

$\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0\}$

some standard sets

$\mathbb{Z}$, $\mathbb{N}$, $\mathbb{Q}$, $\mathbb{R}$ − reals

boolean operations on sets

$\rightarrow$ True False

$A = \{3, 5, 6\}$ $B = \{5, 7, 6, 3\}$

$7 \in A$ — False

$7 \in B$ — True

$B \subseteq B$

$A \subseteq B$ True

$B \subseteq A$ False

$\emptyset \subseteq A$ True $\emptyset \subseteq B$ $\emptyset \subseteq \mathbb{R}$ $\emptyset \subseteq \emptyset$

$A = B$ — requires $A \subseteq B$ and $B \subseteq A$

operations that produce new sets

$A = \{1, 3, 6\}$ $B = \{5, 8, 3\}$ $C = \{17\}$

$A \cap B = \{3\}$ $A \cap C = \emptyset$ $A \cap \emptyset = \emptyset$

$A \cup B = \{1, 3, 6, 5, 8\}$ $A \cup \emptyset = A$

$A \setminus B = A - B = \{1, 6\}$

sets of ~~sets~~. . .and ordered pairs

$A = \{1, 2, 3\} \qquad B = \{g, f\}$

$$A \times B = \{(1, g), (1, f), (2, g), (2, f), (3, g), (3, f)\}$$

$\mathbb{R} \times \mathbb{R}$

$\mathbb{R}^3$

$$\mathcal{P}(B) = \{\{g\}, \{f\}, \emptyset, \{g, f\}\}$$

size of sets

$A = \{1, 2, 3\}$ $B = \{2, 4, 6\}$

$|A| = 3 = |B|$

$|A \cap B| = 1$

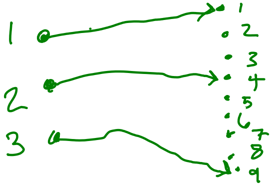
$|A \cup B| = 5$

Combine these ideas

specify functions

$D = \{1, 2, 3\}$ $R = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

- ordered pairs $\{(1,1), (2,4), (3,9)\}$ $(1,7)$ ✗
- pictures



- rule

$f : D \to R$ $f(d) = d^2$

from/to, domain/range, arrow notation

$f : \mathbb{Z} \to \mathbb{N}$ $\quad f(z) = z^2$

one-to-one, onto, etc.

$f(z) = z^2$ not 1-1, e.g. $f(-1) = f(1) = 1$

$f: \mathbb{R} \to \mathbb{R}$, $f(x) = x$

$g: \mathbb{R} \to \mathbb{R}$, $f(x) = x+1$

onto every element of range occurs as target

$f: \mathbb{N} \to 2\mathbb{N}$

$f(x) = 2x$

$A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$

$g: A \to B$, $g(x) = x+1$

g 1-1 ✓

g onto? ✓

$\mathbb{N}$, even $\mathbb{N}$

sums, products

sums

$$1+2+3+\cdots+99+100 = \sum_{i=1}^{i=100} i$$

$$\sum_{i=0}^{i=97} 3+i^2 = \left(\sum_{i=0}^{i=97} 3\right) + \sum_{i=0}^{97} i^2$$

$$\sum_{i=0}^{i=10} 3(i) = 3\sum_{i=0}^{i=10} i$$

$$\sum_{i=0}^{i=5} (i+1) = \sum_{i=1}^{i=6} i$$

$$\prod_{i=1}^{4} i = 1 \times 2 \times 3 \times 4$$

$$\left(\prod_{i=1}^{100} i\right)\left(\prod_{i=1}^{100} \frac{1}{i}\right) = \prod_{i=1}^{i=100} \frac{i}{i} = \prod_{i=1}^{i=100} 1 = 1$$

manipulating sums and products

propositional logic

Statements (closed) evaluate to T, F

3 > 7

- statements, variables – stand for propositions
- operators and, or, not, implies

not ¬, and ∧

$\neg p$

p	$\neg p$
T	F
F	T

truth table

$p \wedge q$

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

or $\vee$, implies $\Rightarrow$

$p \vee q$

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

$R \Leftrightarrow S$

$p \Rightarrow q$

p	q	$p \Rightarrow q$	$\neg p \vee Q$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T

Notes

annotated week 0