PLEASE HAND IN

PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science

Term test #1

CSC 165H1
Duration — 50 minutes

aids allowed: 8.5" x 11" handwritten aid sheet, both sides

Last Name: Sample

First Name: Solution

Do **not** *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below.)

This test consists of 3 questions on 6 pages (including this one). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write "I cannot answer this question," on.

Good Luck!

Question 1. [22 MARKS]

Consider the following python functions and definitions, where `S`, `S1` are sets and `P`, `P1`–`P4` are boolean functions. **Note**: feel free to ask about how python works, since you are being tested on logic, not programming.

```
def q0(S, P):
    return not all({P(x) for x in S})

def q1(S, P):
    return any({P(x) for x in S})

def q2(S, P):
    return not any({P(x) for x in S})

def q3(S, P):
    return all({P(x) for x in S})

S1 = {0, 1, 2, 3, 4, 5}

def P1(x):
    return x > 2

def P2(x):
    return x > 3

def P3(x):
    return P1(x) and P2(x)

def P4(x):
    return (not P2(x)) or P1(x)
```

Part (a) [6 MARKS]

Write the name of each function q0--q3 beside the comment(s) below that best describes the condition for which the function returns True. Indicate which are negations of each other.

1. $\forall x \in S, P(x)$ q3
2. $\exists x \in S, P(x)$ q1
3. $\forall x \in S, \neg P(x)$ q2
4. $\exists x \in S, \neg P(x)$ q0

negations

negations

Part (b) [16 MARKS]

Use your answer for the previous part to predict what the output is below. For each answer, briefly explain your thinking.

1. q0(S1, P2) True

   $\exists x \in S_1, \neg(x>3) \rightarrow$ 0, 1, 2, or 3 are examples.

2. q1(S1, P1) True

   $\exists x \in S_1, x>2 \rightarrow$ sure, 3, 4, 5 are examples.

3. q2(S1, P2) False

   $\forall x \in S_1, \neg(x>3) \longrightarrow$ 4 and 5 are counterexamples

4. q3(S1, P1) False

   $\forall x \in S_1, x>2 \longrightarrow$ 0, 1, 2 are counterexamples

5. q3(S1, P3) False

   $\forall x \in S_1, x>2 \wedge x>3 \longrightarrow$ 0, 1, 2, 3 are counterexamples.

6. q2(S1, P3) False

   $\forall x \in S_1, \neg(x>2 \wedge x>3) \longrightarrow$ 4, 5 are counterexamples

7. q1(S1, P4) True

   $\exists x \in S_1, \neg(x>3) \vee (x>2) \longrightarrow$ 0, 1, 2, 3, 4, 5 are examples.

8. q0(S1, P4) False

   $\exists x \in S_1, \neg(\neg(x>3) \vee (x>2))$

   equivalent $(x>3) \wedge \neg(x>2) \longrightarrow$ no examples.

Question 2. [10 MARKS]

Part (a) [5 MARKS]

Consider the following symbolic statement:

$$S1: \qquad \forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \exists q \in \mathbb{N}, m \times n = q$$

1. Write the negation of the symbolic statement $S1$, in such a way that the negation symbol $\neg$ applies only to predicates such as $m \times n = q$.

(2) $\exists m \in \mathbb{N}, \exists n \in \mathbb{N}, \forall q \in \mathbb{N}, \neg(m \times n = q)$

2. Which is true, statement $S1$ or its negation? Briefly explain your reasoning.

(1) S1 is true

(2) For any $m, n \in \mathbb{N}$, pick $q = m \times n$, which must be a natural number.

Part (b) [5 MARKS]

Now the consider the symbolic statement:

$$S2: \qquad \exists q \in \mathbb{N}, \forall m \in \mathbb{N}, \forall n \in \mathbb{N}, m \times n = q$$

1. Write the negation of the symbolic statement $S2$, in such a way that the negation symbol $\neg$ applies only to predicates such as $m \times n = q$.

(2) $\forall q \in \mathbb{N}, \exists m \in \mathbb{N}, \exists n \in \mathbb{N}, \neg(m \times n = q)$

2. Which is true, statement $S2$ or its negation? Briefly explain your reasoning.

(1) $\neg S2$ is true.

(2) If $q > 0$, pick $m, n = 0$. Otherwise, if $q = 0$ pick $m = n = 1$. Then $m \times n \neq q$

Question 3. [6 MARKS]

Come up with an example of sets D, P, and Q so that one of statements $S3, S4$ is true, and the other is false. Explain which is true, which false, and why.

Interpret $P(x)$ as $x \in P$, $Q(x)$ as $x \in Q$, and $D(x)$ as $x \in D$.

$S3$: $\exists x \in D, (P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x))$ False

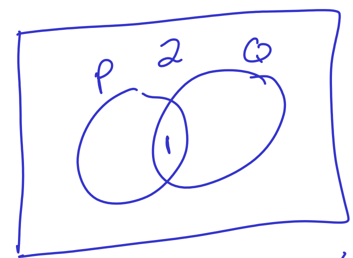
$S4$: $\exists x \in D, \neg P(x) \vee \neg Q(x)$ True

2

$D = \{1, 2\}$

$P = \{1\}$

$Q = \{1\}$



not required

There is no element either in the part of P outside of Q, or the part of Q outside P, so S3 is false. However, the element 2 is outside P, (also outside Q), so S4 is true.

This page is left (nearly) blank to accommodate work that wouldn't fit elsewhere.

1: ______/22

2: ______/10

3: ______/ 6

TOTAL: ______/38