PLEASE HAND IN

UNIVERSITY OF TORONTO
Faculty of Arts and Science

Term test #1

CSC 165H1
Duration — 50 minutes

PLEASE HAND IN

aids allowed: 8.5" x 11" handwritten aid sheet, both sides

Last Name: ______________________________

First Name: ______________________________

Do **not** *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above,
and read the instructions below.)

This test consists of 3 questions on 6 pages (including this one). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write "I cannot answer this question," on.

Good Luck!

Question 1. [22 MARKS]

Consider the following python functions and definitions, where S, S1 are sets and P, P1–P4 are boolean functions. **Note**: feel free to ask about how python works, since you are being tested on logic, not programming.

```
def q0(S, P):
    return any({P(x) for x in S})

def q1(S, P):
    return all({P(x) for x in S})

def q2(S, P):
    return not all({P(x) for x in S})

def q3(S, P):
    return not any({P(x) for x in S})

S1 = {0, 1, 2, 3, 4, 5}

def P1(x):
    return x > 2

def P2(x):
    return x > 3

def P3(x):
    return (not P2(x)) or P1(x)

def P4(x):
    return P1(x) and P2(x)
```

Part (a) [6 MARKS]

Write the name of each function q0--q3 beside the comment(s) below that best describes the condition for which the function returns True. Indicate which are negations of each other.

negation

1. $\forall x \in S, P(x)$ q1
2. $\exists x \in S, P(x)$ q0
3. $\forall x \in S, \neg P(x)$ q3
4. $\exists x \in S, \neg P(x)$ q2

negation

Part (b) [16 MARKS]

Use your answer for the previous part to predict what the output is below. For each answer, briefly explain your thinking.

1. q0(S1, P1) True.

$\exists x \in S_1, x > 2 \longrightarrow$ 3, 4, 5 are examples

2. q1(S1, P2) False

$\forall x \in S_1, x > 3 \longrightarrow$ 0, 1, 2, 3 are counterexamples

3. q2(S1, P1) True

$\exists x \in S_1, \neg(x > 2) \longrightarrow$ 0, 1, 2 are examples.

4. q3(S1, P2) False

$\forall x \in S_1, \neg(x > 3) \longrightarrow$ 4, 5 are counterexamples

5. q0(S1, P3) True

$\exists x \in S_1, \neg(x > 3) \vee (x > 2) \rightarrow$ 0, 1, 2, 3, 4, 5 all examples

6. q1(S1, P3) True

$\forall x \in S_1, \neg(x > 3) \vee (x > 2) \rightarrow$ no counterexamples.

7. q2(S1, P4) True

$\exists x \in S_1, \neg(x > 3 \wedge x > 2) \longrightarrow$ 0, 1, 2, 3 are examples

8. q3(S1, P4) False

$\forall x \in S_1, \neg(x > 3 \wedge x > 2) \longrightarrow$ 4 and 5 are counterexamples

Question 2. [10 MARKS]

Part (a) [5 MARKS]

Consider the following symbolic statement:

$$S1: \qquad \forall m \in \mathbb{N}, \forall n \in \mathbb{N}, \exists q \in \mathbb{N}, m + n = q$$

1. Write the negation of the symbolic statement $S1$, in such a way that the negation symbol $\neg$ applies only to predicates such as $m + n = q$.

② $\exists m \in \mathbb{N}, \exists n \in \mathbb{N}, \forall q \in \mathbb{N}, \neg(m+n=q)$

2. Which is true, statement $S1$ or its negation? Briefly explain your reasoning.

① S1 is true

② For natural numbers m, n, pick $q = m+n \in \mathbb{N}$.

Part (b) [5 MARKS]

Now the consider the symbolic statement:

$$S2: \qquad \exists q \in \mathbb{N}, \forall m \in \mathbb{N}, \forall n \in \mathbb{N}, m + n = q$$

1. Write the negation of the symbolic statement $S2$, in such a way that the negation symbol $\neg$ applies only to predicates such as $m + n = q$.

② $\forall q \in \mathbb{N}, \exists m \in \mathbb{N}, \exists n \in \mathbb{N}, \neg(m+n=q)$.

2. Which is true, statement $S2$ or its negation? Briefly explain your reasoning.

① $\neg S2$ is true

② If $q=0$, pick $m=n=1$. If $q>0$, pick $m=n=0$.

Question 3. [6 MARKS]

Come up with an example of sets D, P, and Q so that one of statements $S3, S4$ is true, and the other is false. Explain which is true, which false, and why.

Interpret $P(x)$ as $x \in P$, $Q(x)$ as $x \in Q$, and $D(x)$ as $x \in D$.

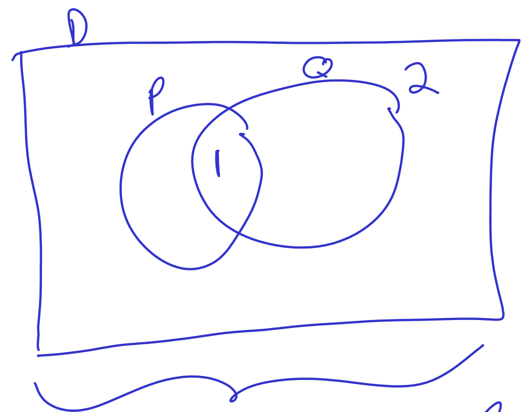
$S3$: $\forall x \in D, P(x) \Leftrightarrow Q(x)$ True

$S4$: $\forall x \in D, P(x) \wedge Q(x)$ False

$D = \{1, 2\}$

$P = \{1\}$

$Q = \{1\}$



not required

There is some element, 2, of D outside $P \cap Q$, so S4 is false. However, there are no elements in P outside Q or Q outside P, so S3 is true.

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1: ________/22

2: ________/10

3: ________/ 6

TOTAL: ________/38