PLEASE HAND IN

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UNIVERSITY OF TORONTO
FACULTY OF ARTS AND SCIENCE

TERM TEST #1

CSC 165H1
DURATION — 50 MINUTES

AIDS ALLOWED: 8.5" x 11" HANDWRITTEN AID SHEET, BOTH SIDES

LAST NAME: Heap

FIRST NAME: Danny

Do NOT *turn this page until you have received the signal to start.*
(In the meantime, please fill out the identification section above, and read the instructions below.)

This test consists of 3 questions on 6 pages (including this one). *When you receive the signal to start, please make sure that your copy of the test is complete.*

Please answer questions in the space provided. You will earn 20% for any question you leave blank or write "I cannot answer this question," on.

Good Luck!

QUESTION 1. [22 MARKS]

Consider the following python functions and definitions, where L is a list and P is a boolean function. NOTE: feel free to ask about how python works, since you are being tested on logic, not programming.

```
def q1(L, P) : return False in [P(x) for x in L]

def q2(L, P) : return False not in [P(x) for x in L]

def q3(L, P) : return True in [P(x) for x in L]

def q4(L, P) : return True not in [P(x) for x in L]

L1 = [0, 1, 2, 3, 4, 5]

def P1(x) : return x > 2

def P2(x) : return x > 3

def P3(x) : return (not P2(x)) or P1(x)
```

$P2(x) \Rightarrow P1(x)$

$x > 3 \Rightarrow x > 2$

PART (A) [6 MARKS]

Write the name of each function q1--q4 beside the comment(s) below that best describes the condition for which the function returns True. Indicate which are negations of each other.

1. $\forall x \in L, P(x)$ q2
2. $\exists x \in L, P(x)$ q3
3. $\forall x \in L, \neg P(x)$ q4
4. $\exists x \in L, \neg P(x)$ q1

negations

negations

PART (B) [16 MARKS]

Use your answer for the previous part to predict what the output is below. For each answer, briefly explain your thinking.

P1 : $x > 2$

1. q1(L1, P1) True — P1(1) is False, so $\exists x \in L1, \neg P1(x)$
2. q2(L1, P1) False — negation of q1
3. q3(L1, P1) True — P1(3) is True, so $\exists x \in L1, P1(x)$
4. q4(L1, P1) False — negation of q3
5. q1(L1, P3) False — negation of q2
6. q2(L1, P3) True $x > 3 \Rightarrow x > 2$ $\forall x \in L1$
7. q3(L1, P3) True — P3(4) is True, so $\exists x \in L1$, P3(x) is True
8. q4(L1, P3) False — negation of q3

QUESTION 2. [10 MARKS]

PART (A) [5 MARKS]

Consider the following symbolic statement:

$$S1: \quad \forall \varepsilon \in \mathbb{R}^+, \exists \delta \in \mathbb{R}^+, \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \varepsilon$$

1. Write the negation of the symbolic statement $S1$, in such a way that the negation symbol $\neg$ applies only to predicates such as $x > \delta$ or $x^3 > \varepsilon$.

$\exists \varepsilon \in \mathbb{R}^+, \forall \delta \in \mathbb{R}^+, \exists x \in \mathbb{R}, x > \delta \wedge \neg(x^3 > \varepsilon)$ either

$\Updownarrow$

$x^3 \leq \varepsilon$

2. Which is true, statement $S1$ or its negation? Briefly explain your reasoning.

S1 is true. Suppose your enemy chooses $\varepsilon > 0$. You then choose $\delta = \sqrt[3]{\varepsilon} > 0$. Then $\forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \delta^3 = (\sqrt[3]{\varepsilon})^3 = \varepsilon$

PART (B) [5 MARKS]

Now the consider the symbolic statement:

$$S2: \quad \exists \delta \in \mathbb{R}^+, \forall \varepsilon \in \mathbb{R}^+, \forall x \in \mathbb{R}, x > \delta \Rightarrow x^3 > \varepsilon$$

1. Write the negation of the symbolic statement $S2$, in such a way that the negation symbol $\neg$ applies only to predicates such as $x > \delta$ or $x^3 > \varepsilon$.

$\forall \delta \in \mathbb{R}^+, \exists \varepsilon \in \mathbb{R}^+, \exists x \in \mathbb{R}, x > \delta \wedge \neg(x^3 > \varepsilon)$

$\Updownarrow$

$x^3 \leq \varepsilon$

2. Which is true, statement $S2$ or its negation? Briefly explain your reasoning.

$\neg S2$ is true. Suppose your enemy chooses δ. You then choose $\varepsilon = 8\delta^3$ and $x = 2\delta$

Then $x = 2\delta > \delta$ and $x^3 = 8\delta^3 \leq \varepsilon$

QUESTION 3. [6 MARKS]

Suppose F is the set of functions, $D(f)$ means "f is differentiable," and $C(f)$ means "f is continuous." Consider the following statement:

$S3$: "Every function is not differentiable unless it is continuous."

Write the contrapositive, converse, and the negation of $S3$ symbolically. NOTE: in this course we translate "unless" as "if not".

contrapositive # $\forall f \in F,\ D(f) \Rightarrow C(f)$

converse # $\forall f \in F,\ \neg D(f) \Rightarrow \neg C(f)$

negation # $\exists f \in F,\ \neg C(f) \wedge D(f)$

This page is left (nearly) blank to accommodate work that wouldn't fit elsewhere.

1: _____/22

2: _____/10

3: _____/ 6

TOTAL: _____/38