

1. Write a detailed, structured proof that

$$\forall f : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}, \forall g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}, g \in \Omega(f) \Rightarrow g^2 \in \Omega(f^2)$$

(where f^2 and g^2 are defined in the obvious way: $\forall n \in \mathbb{N}, f^2(n) = f(n) \cdot f(n)$, and similarly for g).

2. Prove or disprove the following statement:

$$\forall f : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}, \forall g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}, f \in \mathcal{O}(g) \Rightarrow (f + g) \in \Theta(g)$$

(where $(f + g)$ is defined in the obvious way: $\forall n \in \mathbb{N}, (f + g)(n) = f(n) + g(n)$).