Prove or disprove each of the following statements. Write detailed proof structures and justify your work.

1. For all natural numbers n, if there is a natural number k such that $n = 3k + 1$, then there is a natural number j such that $n^2 = 3j + 1$.
2. For all real numbers r, s, if r and s are both positive, then $\sqrt{r} + \sqrt{s} = \sqrt{r + s}$.
3. For all real numbers r, s, if r and s are both positive, then $\sqrt{r} + \sqrt{s} \neq \sqrt{r + s}$.

4. For all real numbers x and y, $x^4 + x^3y - xy^3 - y^4 = 0$ exactly when $x = \pm y$.