1. Write detailed proof *structures* for each of the following statements. **Don't write complete proofs** — for now, focus on the proof structure only and leave out *all* of the actual "content".

(a) $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x \leqslant y \Rightarrow \exists z \in \mathbb{Z}, x \leqslant z \leqslant y$

Assume $x \in \mathbb{Z}$ and $y \in \mathbb{Z}$. # domain assumption
  Assume $x \leqslant y$. # antecedent
    Let $z' = \ldots$ # some value that depends on x and y
    [... proof of $z' \in \mathbb{Z}$...]
    Then, $z' \in \mathbb{Z}$.
    [... proof of $x \leqslant z' \leqslant y$...]
    Then, $x \leqslant z' \leqslant y$.
    Then, $\exists z \in \mathbb{Z}, x \leqslant z \leqslant y$. # introduce $\exists$
  Then, $x \leqslant y \Rightarrow \exists z \in \mathbb{Z}, x \leqslant z \leqslant y$. # introduce $\Rightarrow$
Then, $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x \leqslant y \Rightarrow \exists z \in \mathbb{Z}, x \leqslant z \leqslant y$. # introduce $\forall$

(b) $\forall x \in \mathbb{Z}, (\exists y \in \mathbb{Z}, x = 3y + 1) \Rightarrow (\exists y \in \mathbb{Z}, x^2 = 3y + 1)$

Assume $x \in \mathbb{Z}$. # domain assumption
  Assume $\exists y \in \mathbb{Z}, x = 3y + 1$. # antecedent
    Let y_0 be such that $x = 3y_0 + 1$. # by assumption
    Let $y' = \ldots$ # some value that depends on x and y_0
    [... proof of $y' \in \mathbb{Z}$...]
    Then, $y' \in \mathbb{Z}$.
    [... proof of $x^2 = 3y' + 1$...]
    Then, $x^2 = 3y' + 1$.
    Then, $\exists y \in \mathbb{Z}, x^2 = 3y + 1$. # introduce $\exists$
  Then, $(\exists y \in \mathbb{Z}, x = 3y + 1) \Rightarrow (\exists y \in \mathbb{Z}, x^2 = 3y + 1)$. # introduce $\Rightarrow$
Then, $\forall x \in \mathbb{Z}, (\exists y \in \mathbb{Z}, x = 3y + 1) \Rightarrow (\exists y \in \mathbb{Z}, x^2 = 3y + 1)$. # introduce $\forall$