CSC 165 FALL 2014
TUTORIAL 3

1. Proving equivalence

Suppose P, Q, R, and S are statements.

(1) Prove that $P \Rightarrow (Q \Rightarrow (R \Rightarrow S))$ is equivalent to $(P \wedge Q \wedge R) \Rightarrow S$.
(2) Prove that $((P \Rightarrow Q) \Rightarrow R) \Rightarrow S$ is equivalent to $(\neg P \wedge \neg R) \vee (Q \wedge \neg R) \vee S$.

2. Negation

Negate the following sentences:

(1) Every dog has its day, or perhaps its cat.
(2) $\forall x \in X, \exists y \in Y, x > y \wedge y > x$

3. Guarantees

Consider the statement:

(S1) A and B are both guarantees that C is true.

(1) Write (S1) symbolically. Use parentheses "(" and ")" to make your answer precise.
(2) Choose some appropriate phrases to replace A, B and C. Use these to write (S1) in English. Does this cause you to reconsider your answer to (1)?
(3) Suppose (S1) is true and A is false. What, if anything, can be determined about B and C? Briefly justify.

4. Sample solutions

1. Proving Equivalence

(1) Prove that $P \Rightarrow (Q \Rightarrow (R \Rightarrow S))$ is equivalent to $(P \wedge Q \wedge R) \Rightarrow S$.

$$\begin{aligned} P \Rightarrow (Q \Rightarrow (R \Rightarrow S)) &\Leftrightarrow \neg P \vee (\neg Q \vee (\neg R \vee S)) \quad [\text{transform} \Rightarrow \text{to} \neg \text{ and } \vee] \\ &\Leftrightarrow (\neg P \vee \neg Q \vee \neg R) \vee S \quad [\text{associativity of } \vee] \\ &\Leftrightarrow \neg(P \wedge Q \wedge R) \vee S \quad [\text{DeMorgan's Law}] \\ &\Leftrightarrow (P \wedge Q \wedge R) \Rightarrow S \quad [\text{transform } \neg \text{ and } \vee \text{ to} \Rightarrow]. \end{aligned}$$

(2) Prove that $((P \Rightarrow Q) \Rightarrow R) \Rightarrow S$ is equivalent to $(\neg P \wedge \neg R) \vee (Q \wedge \neg R) \vee S$.

$$\begin{aligned} ((P \Rightarrow Q) \Rightarrow R) \Rightarrow S &\Leftrightarrow \neg(\neg(\neg P \vee Q) \vee R) \vee S \quad [\text{transform} \Rightarrow \text{to} \neg \text{ and } \vee] \\ &\Leftrightarrow ((\neg P \vee Q) \wedge \neg R) \vee S \quad [\text{DeMorgan's Law}] \\ &\Leftrightarrow (\neg P \wedge \neg R) \vee (Q \wedge \neg R) \vee S \quad [\text{distributivity of } \wedge] \end{aligned}$$

5. Negation

(1) Every dog has its day, or perhaps its cat.
Sample solution: Some dog has neither its day nor its cat.

(2) $\forall x \in X, \exists y \in Y, x > y \wedge y > x$
Sample solution: $\exists x \in X, \forall y \in Y, x \leq y \vee y \leq x$

2. Guarantees

(1) $(A \Rightarrow C) \wedge (B \Rightarrow C)$ or $(A \vee B) \Rightarrow C$

(2) "Being rich and being beautiful are both guarantees that one is hated."

(3) Suppose (S1) is true and A is false. What, if anything, can be determined about B and C? Briefly justify.
Nothing. It tells us nothing about C, and A is unrelated to B.

(4) Suppose (S1) is true and C is false. What, if anything, can be determined about A and B? Briefly justify.
A is false and B is false. This comes from the contrapositive(s) of the implication(s), which must be true.