CSC165 tutorial exercise #2
fall 2014
sample solutions

1. "There is no prerequisite for CSC108."

   Sample solution $\forall x \in C, \neg P(x, \text{CSC108})$

2. "Every course has a prerequisite."

   Sample solution $\forall x \in C, \exists y \in C, P(y, x)$

3. "Some course is not a prerequisite for any course."

   Sample solution $\exists x \in C, \forall y \in C, \neg P(x, y)$

4. "No course is a prerequisite for itself."

   Sample solution $\forall x \in C, \neg P(x, x)$

5. "Some courses have several prerequisites."

   Sample solution $\exists x \in C, \exists y \in C, \exists z \in C, P(y, x) \wedge P(z, x) \wedge y \neq z$

6. "No course has more than two prerequisites."

   Sample solution

   $$\forall x \in C, \forall y \in C, \forall z \in C, \forall w \in C, (P(x, w) \wedge P(y, w) \wedge P(z, w)) \Rightarrow (x = y \vee x = z \vee y = z)$$

7. "Some courses have the same prerequisites."

   Sample solution $\exists x \in C, \exists y \in C, \forall z \in C, P(z, x) \Leftrightarrow P(z, y)$

Which are true, which are true in one direction, and which are false both directions? Explain your answers.

1. $\forall x \in D, P(x) \wedge Q(x) \iff (\forall x \in D, P(x)) \wedge (\forall x \in D, Q(x))$

   Sample solution True. Thought of as sets, the left-hand side says that all of D is in the intersection of P and Q, which is the same as saying that all of D is in P and all of D is in Q.

2. $\exists x \in D, P(x) \wedge Q(x) \iff (\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$

Sample solution The left-hand claim implies the right-hand claim. If there is an element of the domain for which P and Q are jointly true, then that same element provides an example where P is true, and the same element provides an example where Q is true. The right-hand claim doesn't imply the left-hand. As a counter-example, consider $D = \mathbb{N}$, $P(n)$: "n is odd", and $Q(n)$: "n is even." In this case the right-hand claim is true: I can find an even natural number, and I can find an odd natural number. However, the left-hand claim is false: I can't find a natural number that is simultaneously even and odd.

3. $\forall x \in D, P(x) \vee Q(x) \iff (\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$

Sample solution The right-hand claim implies the left-hand claim. If the right-hand claim is true, there are two cases to consider. In the first case, P is true of every element of the domain, so it follows that $P \vee Q$ is true of every element of the domain. In the second case, Q is true of every element of the domain, so it follows that $P \vee Q$ is true of every element of the domain. However, the left-hand claim doesn't imply the right-hand claim. As a counter-example, consider (again) $D = \mathbb{N}$, $P(n)$: "n is odd", and $Q(n)$: "n is even." Now the left-hand claim is true, whereas the right-hand claim is false.

4. $\exists x \in D, P(x) \vee Q(x) \iff (\exists x \in D, P(x)) \vee (\exists x \in D, Q(x))$

Sample solution This is true. Thought of as sets, the left-hand side says that the union of P and Q is non-empty, which is true iff P is non-empty or Q is non-empty (which is what the right-hand side says).