CSC165 fall 2014

Mathematical expression

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Course notes, chapter 4

Outline

maximum slice

```
def max_sum(L) :
  """maximum sum over slices of L"""
  max = 0
  i = 0
  while i < len(L) :
    j = i + 1
    while j <= len(L) :
      sum = 0
      k = i
      while k < j :
        sum = sum + L[k]
        k = k + 1
      if sum > max :
        max = sum
      j = j + 1
    i = i + 1
  return max
```

Prove $3n^2 + 2n \in \mathcal{O}(n^2)$

Use $\mathcal{O}(n^2) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2\}$

Special case? what happens if you add a constant?

Prove $3n^2 + 2n + 5 \in \mathcal{O}(n^2)$

Use $\mathcal{O}(n^2) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2\}$

Look at the leading term

Prove: $7n^6 - 5n^4 + 2n^3 \in \mathcal{O}(6n^8 - 4n^5 + n^2)$

Use $\mathcal{O}(6n^8 - 4n^5 + n^2) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq c(6n^8 - 4n^5 + n^2)\}$

how to prove $n^3 \notin \mathcal{O}(3n^2)$?

Negate $\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow n^3 \leq c3n^2\}$

non-polynomials

Big-oh statements about polynomials are pretty easy to prove: $f \in \mathcal{O}(g)$ exactly when the highest-degree term of g is no smaller than the highest-degree term of f.

What about functions such as $\log(n)$ or 3^n? Logarithmic functions are in big-Oh of **any** polynomial, whereas exponential functions (with a base bigger than 1) are not in big-Oh of any polynomial. How do you prove such things?

Prove $2^n \notin \mathcal{O}(n^2)$

Use $\lim_{n \to \infty} 2^n / n^2$

Do you know anything about the ratio $2^n/n^2$, as n gets very large? How do you evaluate:

$$\lim_{n \to \infty} \frac{2^n}{n^2}$$

If the limit evaluates to ∞, then that's the same as saying:

$$\forall c \in \mathbb{R}^+, \exists n' \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n' \Rightarrow \frac{2^n}{n^2} > c$$

Once your enemy hands you a c, you can choose an n' with the required property.

Notes

annotated slides

- monday and friday's annotated slides