CSC165 Fall 2014, Assignment #2

Due November 3rd, 10:00 p.m.

The aim of this assignment is for you to practice devising and presenting proofs. You may work in groups of no more than three students, and you should produce a single solution in a PDF file named `a2.pdf`, submitted to MarkUs.

You will receive 20% of the marks for any question you either leave blank, or write "I cannot answer this." You will receive 0 for any false claim you "prove," or any true claim you "disprove."

1. For $x \in \mathbb{R}$, define $\lfloor x \rfloor$ by:

$$\lfloor x \rfloor \in \mathbb{Z} \wedge \lfloor x \rfloor \leq x \wedge (\forall z \in \mathbb{Z}, z \leq x \Rightarrow z \leq \lfloor x \rfloor).$$

   ... where $\mathbb{Z}$ stands for the set of integers, and $\mathbb{R}$ stands for the set of real numbers. Use the definition of $\lfloor x \rfloor$ to prove or disprove each of the following claims, using the structured proof technique from this course. **Note:** You must use the definition given here, not some other (possibly equivalent) definition for $\lfloor x \rfloor$.

   Claim 1.1:

$$\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, x > y \Rightarrow \lfloor x \rfloor \geq \lfloor y \rfloor.$$

   Claim 1.2:

$$\forall x \in \mathbb{R}, \forall e \in \mathbb{R}^+, \exists d \in \mathbb{R}^+, \forall w \in \mathbb{R}, |x - w| < d \Rightarrow |\lfloor x \rfloor - \lfloor w \rfloor| < e$$

   Claim 1.3:

$$\exists x \in \mathbb{R}, \forall e \in \mathbb{R}^+, \exists d \in \mathbb{R}^+, \forall w \in \mathbb{R}, |x - w| < d \Rightarrow |\lfloor x \rfloor - \lfloor w \rfloor| < e$$

   Claim 1.4:

$$\exists x \in \mathbb{R}, \lfloor x + 1 \rfloor \neq \lfloor x \rfloor + 1$$

2. **Prove** or **disprove** the claim, and **prove** or **disprove** the converse:

   Claim 2.1:

$$\forall n \in \mathbb{N}, (\exists k \in \mathbb{N}, n = 5k + 2) \Rightarrow \left(\exists j \in \mathbb{N}, n^2 = 5j + 4\right)$$

   Claim 2.2:

$$\forall m, n \in \mathbb{N}, (\exists k \in \mathbb{N}, m = 7k + 3) \wedge (\exists j \in \mathbb{N}, n = 7j + 4) \Rightarrow (\exists i \in \mathbb{N}, mn = 7i + 5)$$