CSC165 Fall 2014, Assignment #1

Due October 3rd, 10:00 p.m.

The aim of this assignment is for you to practice using quantifiers, implications, logical connectives and expressions. You may work in groups of no more than three students, and you should produce a single solution in a PDF file named `a1.pdf`, submitted to MarkUs.

You will receive 20% of the marks for any question you either leave blank, or write "I cannot answer this."

1. Suppose A is the set of acronyms, $B(a)$ means a is bifurcated, $C(a)$ means a is catherized, and $D(a)$ means a is diagonal (you do not need to worry about the meaning of acronym, bifurcated, catheterized, or diagonal for the rest of this question). Write the negation of each of the following statements in English, and then in symbolic form.

   (a) All acronyms are catheterized unless they are bifurcated.

   (b) There are no acronyms that are both bifurcated and diagonal.

   (c) All non-diagonal acronyms are catheterized.

   (d) There are diagonal acronyms that are bifurcated.

   (e) For an acronym to be catherized, it is necessary and sufficient that it be diagonal.

2. Consider the sentence:

   S: Every employee who is honest and persistent is successful or bored.

   Each of the statements below is equivalent to either the converse, the contrapositive, or the negation of S. You must decide which label fits each statement, and explain your thinking.

   (a) All employees who are successful or bored must be honest and persistent.

   (b) Every employee who is neither successful nor bored is either dishonest or not persistent.

   (c) Some honest and persistent employee is neither successful nor bored.

   (d) All employees who are dishonest or not persistent must be unsuccessful and not bored.

3. X is a set that contains both students and courses. **Zorn** is a student, and **Zukes** is a course. Predicate $S(x)$ means x is a student, and $C(x)$ means x is a course, $E(x, y)$ means x is enrolled in y, $EQ(x, y)$ means x equals y, and $P(x, y)$ means x is more popular than y. Use set X, **Zorn**, **Zukes**, the predicates above, and the logical connectives you have learned in our course to translate each sentence below, either from symbolic form to English, or from English to symbolic. In English, try to avoid symbols (e.g. x) and predicates (e.g. $C(x)$).

   (a) One, and only one, student in X is more popular than **Zorn**.

(b) $\forall x \in X, (S(x) \wedge E(x, \textbf{Zukes})) \Rightarrow \neg P(x, \textbf{Zorn})$

(c) $\exists x \in X, C(x) \wedge E(\text{Zorn}, x), \forall y \in X, (C(y) \wedge \neg EQ(y, x)) \Rightarrow P(y, x)$

(d) The only course in X that **Zorn** is not enrolled in is **Zukes**

4. For each pair of statements below, given an example of sets D, P, and Q that make one statement true and the other false. Explain the difference in words, and show it with a Venn diagram.

   (a) The pair $\forall d \in D, P(d) \Rightarrow Q(d)$ and $\forall d \in D, P(d) \wedge Q(d)$.

   (b) The pair $\exists d \in D, P(d) \wedge Q(d)$ and $\exists d \in D, P(d) \Rightarrow Q(d)$.

5. Suppose T is a set of natural numbers, and we have the following statement about T:

   S2: Every element of T is 2 raised to some natural number power.

   Which of the following statements imply $S2$? Which of the following statements are implied by $S2$? Explain.

   (a) $T = \{64, 32, 128\}$.

   (b) If $i, j \in T$, and $i < j$, then i divides j.

   (c) T has no more than 1 odd member.

   (d) No odd prime number divides any element of T.

   (e) T is the empty set.