CSC 165

bounds and problems

week 9, lecture 3

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Reminder A2 is due this week

- Today's tutorial focuses on proof/disproof structure
- special virtual Monday tutorial on using LaTeX (optional) BA 2200 6-8 on Wednesday

re-marks This week ...

recall the problem

You can get a handout to work on in groups. Here's the gist of the problem:

Suppose you're given the list:

37	93	0	23	79	65	49	81	67	8	32	29	96	76	15	9	51	14	29	69

promising exponential solution

Can you find one or more longest non-decreasing sequences?

For example, 37, 93, 96 is a sequence that's non-decreasing,

but you can easily find longer ones.

Sequences are ordered, but need not be contiguous

Can you find the length of an longest sequence ending at index something.

State of the art, so far: focus on the longest non-decreasing sequence ending at a particular index

$f(n) = \frac{1}{n}$
not defined when $n=0$

Let $\mathcal{F} = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0}\}$

$f(n) = \begin{cases} 1/n & n>0 \\ 17\tfrac{1}{2} & n=0 \end{cases}$

Is it true that $\forall f, g \in \mathcal{F}, \boxed{f \in \mathcal{O}(g) \Rightarrow f \in \mathcal{O}(g^2)}$?

$g \in O(g^2)$?

$g(n) = f(n)$

what if denominator of g increase?

Suppose I found $c \in \mathbb{R}^+$, $B \in \mathbb{N}$
st $\forall n \in \mathbb{N}$, $n \geq B \Rightarrow f(n) \leq c\,g(n)$

Claim $\forall f, g \in \mathcal{F}$, $f \in O(g) \Rightarrow f^2 \in O(g^2)$

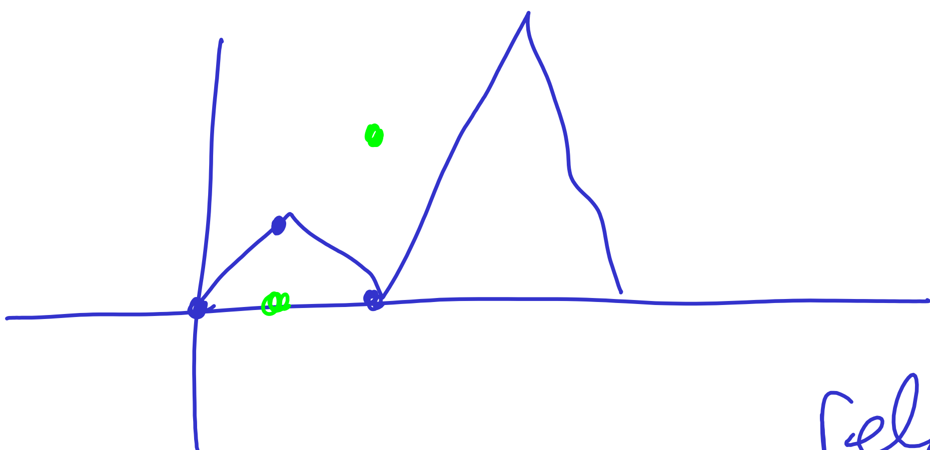
Suppose I have $c \in \mathbb{R}^+$, $B \in \mathbb{N}$, st
$\forall n \in \mathbb{N}$, $n \geq B$, $f(n) \leq c\,g(n)$
$\Longrightarrow$ $f^2(n) \leq c\,g(n)\,f(n)$
$\leq [c\,g(n)]^2$

Is it true that $\forall f, g \in \mathcal{F}, f \in \mathcal{O}(g) \vee g \in \mathcal{O}(f)$?

No – what if f and g are periodic. (trig functions)

let $f(n) = \begin{cases} 0 & n \text{ even} \\ n & n \text{ odd} \end{cases}$

$g(n) = \begin{cases} n & n \text{ even} \\ 0 & n \text{ odd.} \end{cases}$



exercise

prove $f(n) \notin \Theta(g)$

and $g \notin \Theta(f)$

related question – when can we bound $g(n) \equiv c$

some calculus

$e^{\ln x} = 2^{\log_2 x}$

$\ln x = \ln 2 \cdot \log x$

$\Rightarrow \frac{\ln x}{\ln 2} = \log x$

How do you evaluate

$$\lim_{x\to\infty} \frac{x}{\log x} = \lim_{x\to\infty} \frac{x'}{(\log x)'}$$

$$= \lim_{x\to\infty} \frac{1}{\frac{1}{x} c} = \lim_{x\to\infty} \frac{x}{c} = \infty$$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$$

try L'Hôpital's rule (the limit of the derivatives)
what does it mean?

Use this to show $n \notin O(\log x)$

$\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, n \geq B \wedge n > c \cdot \log n$

If the limit of the ratio is infinity, then for every $x_1 \in \mathbb{R}^+$,
$\exists x_2 \in \mathbb{R}^+, \forall x \in \mathbb{R}, x \geq x_2 \Rightarrow \frac{x}{\log x} > x_1$

$\max \lceil x \rceil, B$

$\lceil x \rceil > x_1 \cdot \log x$

$x_1 = c$