CSC 165

bounds

week 9, lecture 1

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We've proved: $P(n) : 2^n \geq 2n$

Use this to prove that $2^n \notin \mathcal{O}(n)$

scratch

bounded below

Notice that the definition of big-Omega differs in just one character from big-Oh:

$\Omega(g) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \geq cg(n)\}$

The rôle of B is, as with big-Oh, to act as a breakpoint, so comparisons don't have to start at the origin

The rôle of c is to scale g *down* below f.

If you're proving $f \in \Omega(g)$, you get to choose c and B to suit your proof.

one last bound

It often happens that functions are bounded above *and* below by the same function. In other words, $f \in \mathcal{O}(G) \wedge f \in \Omega(g)$. We combine these two concepts into $f \in \Theta(g)$.

$$\Theta(g) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c_1 \in \mathbb{R}^+, \exists c_2 \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n)\}$$

You might want to draw pictures, and conjecture about values of c_1, c_2, B for $f = 5n^2 + 15$ and $g = n^2$.

some theorems

How do you deal with a general statement about two functions:

$(f \in \mathcal{O}(g) \wedge g \in \mathcal{O}(h)) \Rightarrow f \in \mathcal{O}(h)$

scratch

How about: $g \in \Omega(f) \Leftrightarrow f \in \mathcal{O}(g)$

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