CSC 165

polynomials

week 8, lecture 2

Danny Heap

heap@cs.toronto.edu

www.cdf.toronto.edu/~heap/165/F09

pinning down intuition

We know, or have heard, that polynomials of the same degree grow at "roughly" the same speed

We want to make this "roughly" explicit

Here's how we define $\mathcal{O}(n^2)$, functions that eventually grow no more quickly than n^2

$$\mathcal{O}(n^2) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cn^2\}$$

The definition says that there's a multiplier, c, such that if you go far enough to the right, B, the graph of f is bounded above by the graph of cn^2

Prove: $3n^2 + 2n \in \mathcal{O}(n^2)$

scratch

in general, $\mathcal{O}(g)$:

$$\mathcal{O}(g) = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0} \mid \exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \Rightarrow f(n) \leq cg(n)\}$$

Prove: $7n^6 - 5n^4 + 2n^3 \in \mathcal{O}(6n^8 - 4n^5 + n^2)$

scratch

how to prove $n^4 \notin \mathcal{O}(3n^2)$

begin by stating the negation, then prove that

scratch