CSC 165

non-boolean functions

week 7, lecture 2

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office hour: how about FM @ 1:30

post test & assignment: when Gary passes them on, I'll carry them to lecture (for a while).

coursework modifications

Three imminent pieces of coursework aren't ready for your attention yet. Let's vote on:

A: Replace exercises 5 and 6 by exercises 3 and 4, delay assignment 2 For – 37
rationale: the original exercises 3 and 4 were due yesterday,
and assignment 2 next week, and you need reasonable time to do them. Against – 0

B (if A passes): spread the 6% weight of the missing two exercises uniformly For 6
over all other pieces of course work (2 tests, 4 exercises, 3 assignments, tutorials), Against
increasing each weight by 0.6% ~~8~~ 14

C vs D 19 + tutorial For 20

C (compare to B): spread the 6% weight of the two missing exercises
uniformly over all future term work (1 test, 2 assignments, 2 exercises) tutorials
increasing each weight by $\frac{6}{7}\%$ ~~4%~~ 1% ↓ including Against – 6

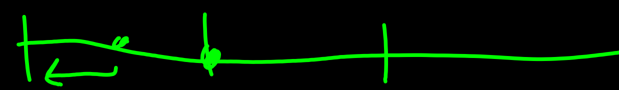
D (compare to B and C): spread the 6% weight of the two missing exercises For 11
uniformly over future exercises and assignments, increasing each weight by 1.5% Ag 8

D vs C 14

scratch

be careful quantifying non-booleans

we're used to functions and conditions such as "odd," "even", $>$ returning boolean (true, false) results, and combining them with quantifiers such as $\forall$ or $\exists$



what about functions that return other values
— natural numbers, real numbers, for example?

$y \in \mathbb{R}$

computer scientists often use an innocent-looking function $\lfloor x \rfloor$, meaning:
$y = \lfloor x \rfloor$ if and only if $y \in \mathbb{Z} \wedge y \leq x \wedge (\forall z \in \mathbb{Z}, z \leq x \Rightarrow z \leq y)$

as a warm-up, use the definition to prove that $\lfloor x \rfloor$ is always less than $x + 1$.

larger!

$$\forall x \in \mathbb{R}, \lfloor x \rfloor < x + 1$$ Prove

$$y = \lfloor x \rfloor \text{ if and only if } y \in \mathbb{Z} \wedge y \leq x \wedge (\forall z \in \mathbb{Z}, z \leq x \Rightarrow z \leq y)$$

NOT! $\forall \lfloor x \rfloor \in \mathbb{R} \ldots$

how many parts of the definition of $\lfloor x \rfloor$ did you need in previous proof? For more challenge, try:

$$\forall x \in \mathbb{R}, \lfloor x \rfloor > x - 1$$

$$y = \lfloor x \rfloor \text{ if and only if } y \in \mathbb{Z} \wedge y \leq x \wedge (\forall z \in \mathbb{Z}, z \leq x \Rightarrow z \leq y)$$

y x

$z \leq y$

Assume $x \in \mathbb{R}$ # x is generic

Let $y = \lfloor x \rfloor$

Then $y \leq x$ # by defn

$(\forall z \in \mathbb{Z}, z \leq x \Rightarrow z \leq y)$ # by defn

~~and~~ & # from defn.

$y \in \mathbb{Z}$

Then $y+1 \in \mathbb{Z} \wedge y+1 > y$ # by contrapos

Then $y+1 > x$

Then $y > x-1 \wedge y = \lfloor x \rfloor$ # re-arrange

Then $\lfloor x \rfloor > x-1$

Conclude $\forall x \in \mathbb{R}, \lfloor x \rfloor > x-1$ # shown uni

slide 6

proof by cases

You can prove by induction (CSC236) that:

$$\forall m \in \mathbb{N}\, \forall n \in \mathbb{N}, n \neq 0 \Rightarrow \exists! q \in \mathbb{N}, \exists! r \in \mathbb{N}, m = qn + r \text{ and } n > r \geq 0$$

$\exists!$ is a compact way of saying there exists exactly one.

q and r are called the quotient and remainder, respectively. We also denote r by $m \bmod n = r$

A consequence is that any natural number n has a remainder of either 0, 1, or 2 after division by 3. What possible remainders are there for perfect squares after division by 3?

Would you believe:

$$\forall n \in \mathbb{N}, n^2 \bmod 3 \neq 2$$

Prove: $\forall n \in \mathbb{N}, n^2 \bmod 3 \neq 2$

I think you'll need cases for different possible results of $n \bmod 3$