CSC 165

indirect proof

Danny Heap

heap@cs.toronto.edu

www.cdf.toronto.edu/~heap/165/F09

implication not symmetrical

- proving that $p \Rightarrow q$ involves finding a chain of intermediate results

  $p \Rightarrow p_1 \Rightarrow \cdots \Rightarrow p_n \Rightarrow q$

  what happens if your search is unlucky?

- try reversing the search, use the contrapositive

  recall that $p \Rightarrow q$ is true exactly when $\neg q \Rightarrow \neg p$

  proving one proves the other

- for example, how would you go about proving

  $\forall n \in \mathbb{N}, n^2$ odd $\Rightarrow n$ odd

direct proof difficult here

- You could get python to verify this for lots of natural numbers

  for n in range(0,1000) :

  n * n % 2 == 0 or n % 2 == 1

  but that's pretty lame

- or, imitate the direct proof of the converse:

  Assume $n \in \mathbb{N}$,

  Assume n^2 is odd. Then $\exists k \in \mathbb{N}, n^2 = 2k + 1$ # definition of odd

  hmmm. . . should we take the square root of $2k + 1$ or what????

Prove $\forall n \in \mathbb{N}, n^2$ odd $\Rightarrow n$ odd

same as: Prove $\forall n \in \mathbb{N}, \neg n$ odd $\Rightarrow \neg n^2$ odd

scratch

getting contradictory

- what happens if you want to prove q,
  so you'd like some well-known p to imply q,
  but you can't decide which p is right for the job?

- why not just take the entire sum of well-known facts as your antecedent
  $p_0 \wedge p_1 \wedge p_2 \wedge \cdots \wedge p_n \Rightarrow q$

- how does this help? this is equivalent to saying that
  $\neg q$ implies that some well-known fact is false — contradiction!
  $\neg q \Rightarrow \neg p_0 \vee \neg p_1 \vee \neg p_2 \vee \cdots \vee \neg p_n$

there are infinitely many prime numbers

- $P = \{n \in \mathbb{N} \mid n \text{ has exact 2 factors in } \mathbb{N}\}$
  Claim SP: $\forall n \in \mathbb{N}, |P| > n$. Prove by contradiction.

scratch

format of contradiction

- Assume $\neg q$

  within the assumption, follow a chain of implications

  $\vdots$

  arrive at a contradiction of some already-known fact

  Conclude q, since assuming $\neg q$ led to a contradiction.

coursework proposal

- look over the proposed course calendar at

  www.cdf.toronto.edu/~heap/165/F09
  and be prepared to vote on Friday October 23rd

scratch