CSC 165

truncation

week 12, lecture 3

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resources: chapter 7 of course notes

http://docs.python.org/tutorial/floatingpoint.html

http://en.wikipedia.org/wiki/IEEE_754-2008

approximating functions

Many important functions are approximated by using part of their Taylor series expansion:

$e^x =$ $\exp(x)$

or $\sin(x) =$

$$e^x = 1 + x + x^2/2! + \frac{x^3}{3!} + \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Calculus provides a bound on how much information you lose by truncation, and now you've got truncation and rounding as possible sources of error.

parsing the blame

We want to apply the exact function, f, to an exact value x, yielding $f(x)$

We settle for an approximate function $\hat{f}$ applied to an approximate value x', yielding $\hat{f}(x')$

total error

We can break up the difference, $|\hat{f}(x') - f(x)|$, into two parts,
to account for the source of the error:

$$|\hat{f}(x') - f(x)| = |\hat{f}(x') - f(x') + f(x') - f(x)|$$

$$\le |\hat{f}(x') - f(x')| + |f(x') - f(x)|$$

3 terms vs ∞ terms.