CSC 165

condition

week 12, lecture 2

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resources: chapter 7 of course notes

http://docs.python.org/tutorial/floatingpoint.html

diminishing errors

We saw that the quadratic formula had two opportunities to experience
catastrophic cancellation, both involving the parameter b in

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What's the situation when $b = 0$?

$$\frac{\sqrt{-4ac}}{2a}$$

This ends up being no worse than the square root operation,
even if we have to deal with $i = \sqrt{-1}$. What does that do to input errors?

square root squashes error

Suppose we calculate $\sqrt{c}$ where the true value of $c = 0.25$,
but we calculate with a poor approximation $c' = 0.36$.

The relative error of the input is $|0.25 - 0.26|/|0.25| = 0.11/0.25 = 44\%$
The relative error of the output is $|0.5 - 0.6|/|0.5| = 0.1/0.5 = 20\%$
Taking the square root halved the relative error!

This isn't a fluke due to some special choice of $c = 0.25$ and $c' = 0.36$.
Do the algebra to work out the general case, and square root always reduces relative error.

scratch

condition number

The relative error of the output over the relative error of the input
is an important enough concept to have a name: *condition number*

What is the limiting behavior of the condition number as errors get very small?

what the condition number means

What's the condition number for $f(x) = x^5$? How about $f(x) = \cos(x)$?
What does this tell you about algorithms to implement f in certain regions?

subsequence problem

How many times does the string AB occur as a subsequence of $ACBCAC$?

In general, how do you count the number of times string1
occurs as a subsequence of string2?